

# Teacher Energized Resource Manual

**Class : 9<sup>th</sup>**  
**Subject : Mathematics**



CENTRAL BOARD OF SECONDARY EDUCATION

## Preface

In consonance with the move towards outcome-based education where focus is on developing competencies in students, the Central Board of Secondary Education is delighted to share the *Teacher Energized Resource Manual* that will aid teachers in aligning their classroom transaction to a competency framework.

Each chapter of the Resource Manual corresponds to the respective chapters in the NCERT textbooks. The chapters have been chunked by concept; these concepts have been linked to the NCERT Learning Outcomes; and an attempt has been made to delineate Learning Objectives for each concept. Every chapter has a set of assessment items, where two items have been provided as examples for each Learning Objective. Teachers can use these to assess if the learner has acquired the related concept. Needless to say, the items are illustrative examples to demonstrate how competency-based items can be prepared to measure Learning Objectives and Outcomes. The variety in item forms is suggestive of the ways in which a particular concept can be assessed to identify if the learner has attained different competencies. We trust and hope that teachers would be able to generate many more similar test items for use in practice.

Your observations, insights and comments as you use this Resource Manual are welcome. Please encourage your students to voice their suggestions as well. These inputs would be helpful to improve this Manual as these are incorporated in the subsequent editions. All possible efforts have been made to remove technical errors and present the Manual in a form that the teachers would find it easy and comfortable to use.

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## HOW TO USE THIS MANUAL

The goal of the Teacher Energized Resource Manual (TERM) is to provide teachers with competency-based education resources aligned to NCERT textbooks that would support them in the attainment of desired Learning Outcomes and development of requisite competencies of the learner. The TERM has equal number of corresponding chapters as NCERT Textbooks with listing of Concepts, Learning Outcomes developed by NCERT and Learning Objectives. Competency based test items for each corresponding Learning Objective and sample activities for enrichment have been provided.

### **Learning Objectives:**

Each chapter has a *Learning Objectives* table. The table also lists the Concepts covered in the chapter. Learning Objectives are broken down competencies that a learner would have acquired by the end of the chapter. They are a combination of skills and what the learner would use this skill for. For example, the first Learning Objective in the table below relates to the skill of *application* and the students will use this competency to obtain the highest common factor of 2 positive integers. Teachers can use these specific Learning Objectives to identify if a student has acquired the associated skill and understands how that skill can be used.

Concepts	Learning Objectives	Learning Outcomes
Euclid's Division	Apply Euclid Division Algorithm in order to obtain HCF of 2 positive integers in the context of the given problem	Generalises properties of numbers and relations among them studied earlier, to evolve results, such as, Euclid's division algorithm, fundamental theorem of arithmetic in order to apply them to solve problems related to real life contexts
	Apply Euclid Division Algorithm in order to prove results of positive integers in the form of $ax+by$ where a and b are integers	
Fundamental Theorem of Arithmetic	Use the Fundamental Theorem of Arithmetic in order to calculate HCF and LCM of the given numbers in the context of the given problem	
Irrational Numbers	Recall the properties of irrational number in order to prove that whether the sum/difference/product/quotient of 2 numbers is irrational or not	
	Apply theorems of irrational number in order to prove whether a given number is irrational or not	
Decimal Representation of Irrational Numbers	Apply theorems of rational numbers in order to find out about the nature of their decimal representation and their factors	

### **Concepts:**

The important concepts in a particular chapter are listed in the first section. Most often, they follow a logical order and present a sequence in which these are likely to be covered while teaching. In case, your teaching strategy is different and presents them in a different order, you need not worry. Teach the way, you consider the best. You only need to ensure their understanding and the attainment of desired learning objectives.

### **Learning Outcomes (NCERT):**

A mapping of Learning Outcomes developed by the NCERT and Learning Objectives is provided in last column of the table. The Learning Outcomes have been developed by the NCERT. Each Learning Objective is mapped to NCERT Learning Outcomes and helps teachers to easily identify the larger outcome that a learner must be able to demonstrate at the end of the class/ chapter.

**Test items:**

For each Learning Objective, at least two competency-based test items have been provided. Although, the items in this resource manual are multiple choice questions, which assess developed competencies of a student rather than only knowledge, it must be kept in mind that there can be different kinds of assessment that can easily align with competency-based education. Teachers can use these items to assess if a learner has achieved a particular Learning Objective and can take necessary supportive actions. Teachers are also encouraged to form similar questions which assess skills of students.

<p><b>LOB:</b> Apply Euclid Division Algorithm in order to obtain HCF of 2 given numbers in the context of the given problem</p> <p>I. A worker needs to pack 350 kg of rice and 150 kg of wheat in bags such that each bag weighs the same. Each bag should either contain rice or wheat. Which option shows the correct steps to find the greatest amount of rice/wheat the worker can pack in each bag?</p> <p><b>Option 1:</b> Step 1: <math>350 = 2(150) + 50</math> Step 2: <math>150 = 3(50) + 0</math> Step 3: Greatest amount: 50 kg</p> <p><b>Option 2:</b> Step 1: <math>350 = 2(150) + 50</math> Step 2: <math>150 = 2(50) + 0</math> Step 3: Greatest amount: 50 kg</p> <p><b>Option 3:</b> Step 1: <math>350 = 2(150) + 50</math> Step 2: <math>150 = 3(50) + 0</math> Step 3: Greatest amount: 150 kg</p> <p><b>Option 4:</b> Step 1: <math>350 = 2(150) + 50</math> Step 2: <math>150 = 2(50) + 0</math> Step 3: Greatest amount: 150 kg</p> <p><b>Correct Answer:</b> Option 1</p>
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Fig: 3

**Suggested Teacher Resources**

At the end of each chapter, certain activities have been suggested which can be carried out by the teachers with learners to explain a concept. These are only samples and teachers can use, adapt, as well as, create activities that align to a given concept.

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# I. NUMBER SYSTEMS

**QR Code:**



**Learning outcome and Learning Objectives:**

Content Area/ Concepts	Learning Objectives	Learning Outcome
Introduction to number system	Recall natural numbers, whole numbers, integers and Rational numbers in order to classify a given number as either of them	Applies logical reasoning in classifying real numbers, and proving their properties in order to use them in different situations
Irrational Numbers	Represent a given number in the form $p/q$ in order to show whether the given number is rational or not	
	Calculate and find rational numbers between any 2 rational numbers in order to prove that there are infinite rational numbers between any 2 given rational numbers	
	Modify a given non-terminating decimal number in the form of $p/q$ in order to comment whether this number is irrational	
Real Numbers and their Decimal Expansions	Use Pythagoras' theorem and create a Pythagorean triplet in order to construct the length of root of a given number	
	Deduce the value of a given fraction in its decimal form in order to infer if the decimal number is terminating or non-terminating	
Representing Real Numbers on the Number Line	Use successive magnification in order to visualise a given decimal number on a number line	
Operations on Real Numbers	Compute the commutative, associative and distributive laws for addition and multiplication for irrational numbers in order to determine whether the sum, difference, quotients and products of irrational numbers are irrational or not	
	Rationalize the denominator of a given expression with a square root term in the denominator in order to convert it to an equivalent expression whose denominator is a rational number	
Laws of Exponents for Real Numbers	Extend the law of exponents in order to simplify a given expression	



# Test items



**LOB:** Recall natural numbers, whole numbers, integers and Rational numbers in order to classify a given number as either of them

1. Which statement is NOT true?
  - a. 11 is an integer
  - b. -5 is a whole number
  - c. 8 is a natural number
  - d. 3 is a rational number

**Correct Answer:** Option b

2. A number is of the form  $-q/2$ . Given that it is a whole number, an integer and a rational number, which of these is true?
  - a.  $q < 0$  and the digit in the unit place of  $q$  is 2, 4, 6, 8, 0.
  - b.  $q > 0$  and the digit in the unit place of  $q$  is 2, 4, 6, 8, 0.
  - c.  $q < 0$  and the digit in the unit place of  $q$  is 1, 3, 5, 7, 9.
  - d.  $q > 0$  and the digit in the unit place of  $q$  is 1, 3, 5, 7, 9.

**Correct Answer:** Option a

**LOB:** Represent a given number in the form  $p/q$  in order to show whether the given number is rational or not

1. A student represents an integer 8 as  $8/1$ . Which of the following is true about the integer?
  - a. It is a rational number as it is in the form  $p/q$ , where  $q \neq 0$ .
  - b. It is an irrational number as it is in the form  $p/q$ , where  $q \neq 0$ .
  - c. It is not a rational number as rational number is in the form  $p/q$ , where  $q \neq 1$ .
  - d. It is not an irrational number as irrational number is in the form  $p/q$ , where  $q \neq 1$ .

**Correct Answer:** Option a

2. Is -10 a rational number? Why or why not?
  - a. No, as  $-10 = -10/0$  and rational numbers are ratio of integers  $m$  and  $n$ , where  $n \neq 0$ .
  - b. Yes, as  $-10 = -10/1$  and rational numbers are ratio of integers  $m$  and  $n$ , where  $n \neq 0$ .
  - c. No, as  $-10 = -10/1$  and rational numbers are ratio of integers  $m$  and  $n$ , where  $n \neq 1$ .
  - d. Yes, as  $-10 = -10/0$  and rational numbers are ratio of integers  $m$  and  $n$ .

**Correct Answer:** Option b

**LOB:** Calculate and find rational numbers between any 2 rational numbers in order to prove that there are infinite rational numbers between any 2 given rational numbers

1. Which rational number lies between  $1/3$  and  $3/4$ ?
  - a.  $2(1/3 + 3/4)$
  - b.  $2(1/3 - 3/4)$
  - c.  $1/2(1/3 + 3/4)$
  - d.  $1/2(1/3 - 3/4)$

**Correct Answer:** Option c

2. Some of the rational numbers between 7 and 11 can be expressed in the form  $m/6$ , where  $m$  belongs to a set of natural numbers. Which of the following statement is true?
  - a. All possible values of  $m$  lie between 42 and 66.
  - b. All possible values of  $m$  lie between 42 and 77
  - c. All possible values of  $m$  lie between 48 and 60.
  - d. All possible values of  $m$  lie between 66 and 77.

**Correct Answer:** Option a

**LOB:** Modify a given non-terminating decimal number in the form of  $p/q$  in order to comment whether this number is irrational

1. Which of the following is true about  $x = 0.\bar{6}$ ?

- a.  $x$  is a rational number because  $x$  can be written in the form  $p/q$  by solving the equation  $10x = 6 + x$ .
- b.  $x$  is a rational number because  $x$  can be written in the form  $p/q$  by solving the equation  $10x = 6 - x$ .
- c.  $x$  is an irrational number because  $x$  can be written in the form  $p/q$  by solving the equation  $10x = 6 + x$ .
- d.  $x$  is an irrational number because  $x$  can be written in the form  $p/q$  by solving the equation  $10x = 6 - x$ .

**Correct Answer:** Option a

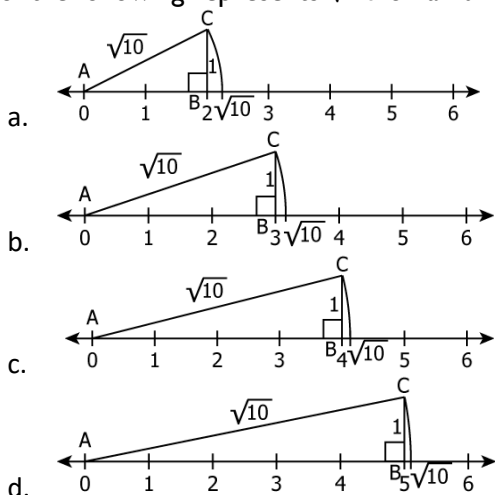
2. Which of these is equivalent to  $0.5\overline{782}$ ?

- a.  $5772/9990$
- b.  $5777/9990$
- c.  $5782/9990$
- d.  $5787/9990$

**Correct Answer:** Option b

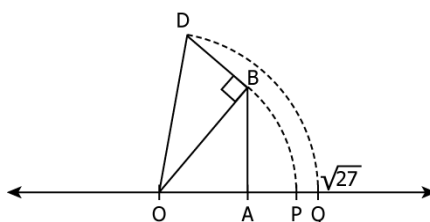
**LOB:** Use Pythagoras' theorem and create a Pythagorean triplet in order to construct the length of root of a given number

1. Which of the following represents  $\sqrt{10}$  on a number line?



**Correct Answer:** Option b

2. Kevin's work to represent 27 on a number line is shown. In the number line, arc DQ is drawn using OD as the radius.



Looking at Kevin's work, Tina and Ajay made following statements.

Tina:  $OA = 5$  units,  $AB = BD = 1$  unit

Ajay:  $OB = 26$  units and  $AB = 1$  unit

Who is correct?

- a. Only Tina
- b. Only Ajay
- c. Both of them
- d. Neither of them

**Correct Answer:** Option c

**LOB:** Deduce the value of a given fraction in its decimal form in order to infer if the decimal number is terminating or non-terminating

1. Which of the following rational numbers is equivalent to a decimal that terminates?

- a.  $1/3$

- b.  $\frac{3}{8}$
- c.  $\frac{5}{6}$
- d.  $\frac{8}{9}$

**Correct Answer:** Option b

2. A rational number  $m$  is described below.  
 $m = \text{Prime number} / 15$

Consider the following conclusions about the number  $m$ .

Conclusion 1: The decimal expansion of  $m$  is non-terminating recurring but the decimal expansion of  $3m$  is terminating.

Conclusion 2: The decimal expansion of  $m$  is terminating.

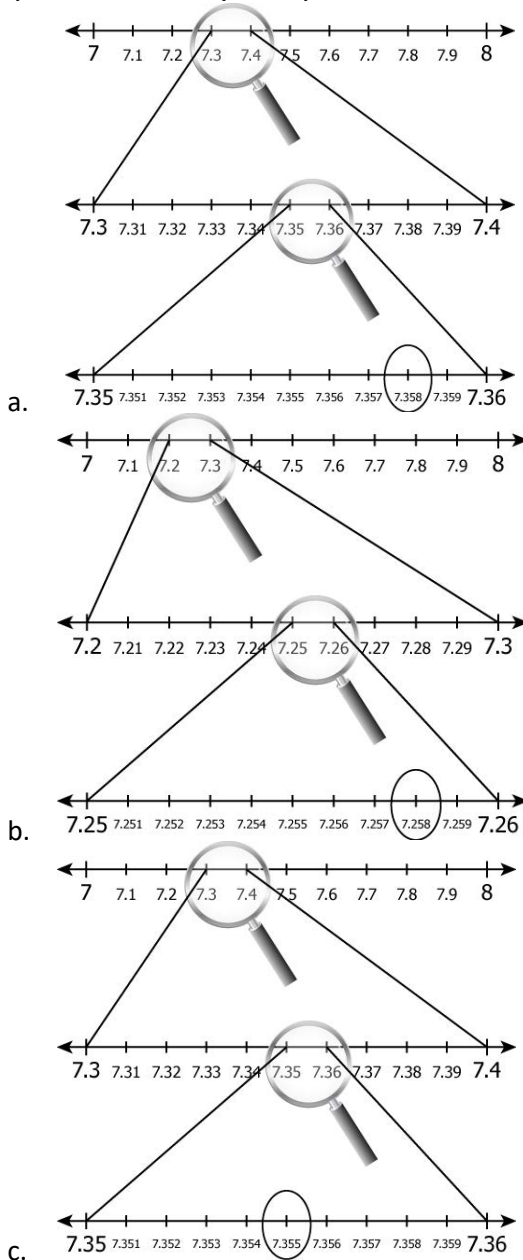
Which of these is correct?

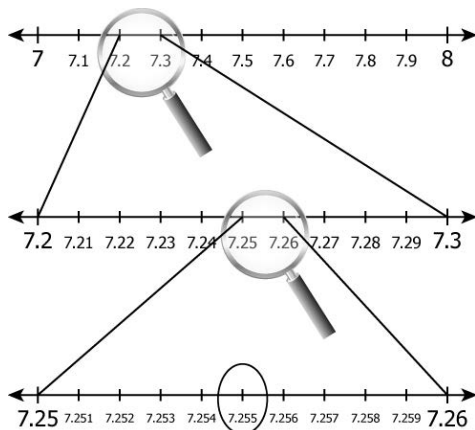
- a. Conclusion 1 is valid but Conclusion 2 is valid only for the prime number 3.
- b. Conclusion 1 is valid but Conclusion 2 is valid only for the prime number 5.
- c. Conclusion 1 is not valid but Conclusion 2 is valid only for the prime number 3.
- d. Conclusion 1 is not valid but Conclusion 2 is valid only for the prime number 5.

**Correct Answer:** Option a

**LOB:** Use successive magnification in order to visualise a given decimal number on a number line

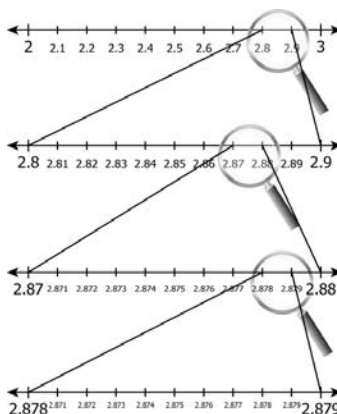
1. Which option shows a way to represent 7.358 on a number line?





d. **Correct Answer:** Option a

2. Tina uses a magnifying glass to find a decimal 2.8785 on a number line as shown. Tina first looks between 2.8 and 2.9. She then selects



Between which two numbers does the decimal 2.87 lie?

- a. 2.8785 and 2.8786
- b. 2.8786 and 2.8787
- c. 2.8787 and 2.8788
- d. 2.8788 and 2.8789

**Correct Answer:** Option c

**LOB:** Compute the commutative, associative and distributive laws for addition and multiplication for irrational numbers in order to determine whether the sum, difference, quotients and products of irrational numbers are irrational or not

1. Which of the following is an irrational number?
  - a.  $\sqrt{12}/\sqrt{3}$
  - b.  $\sqrt{18}/\sqrt{2}$
  - c.  $\sqrt{45}/\sqrt{5}$
  - d.  $\sqrt{42}/\sqrt{7}$

**Correct Answer:** Option d

2. An expression is given:  $2(\sqrt{k-1}) + \sqrt{8}$   
 If on adding  $-8\sqrt{2}$  to the expression results in a rational number, what is the value of k?
  - a. 6
  - b. 12
  - c. 18
  - d. 36

**Correct Answer:** Option c

**LOB:** Rationalize the denominator of a given expression with a square root term in the denominator in order to convert it to an equivalent expression whose denominator is a rational number

1. Which of these is a way to convert  $\frac{15}{\sqrt{63+\sqrt{20}}}$  to an equivalent number whose denominator is a rational number?

- a.  $\frac{15}{\sqrt{63+\sqrt{20}}} \times \frac{(3\sqrt{7}-2\sqrt{5})}{(3\sqrt{7}-2\sqrt{5})}$
- b.  $\frac{15}{\sqrt{63+\sqrt{20}}} \times \frac{(3\sqrt{7}-4\sqrt{5})}{(3\sqrt{7}-4\sqrt{5})}$
- c.  $\frac{15+(3\sqrt{7}-4\sqrt{5})}{(\sqrt{63+\sqrt{20}}) \times (3\sqrt{7}-4\sqrt{5})}$
- d.  $\frac{15+(3\sqrt{7}-2\sqrt{5})}{(\sqrt{63+\sqrt{20}}) \times (3\sqrt{7}-2\sqrt{5})}$

**Correct Answer:** Option a

2. On a number line,  $\frac{2}{\sqrt{8}}$  is halfway located between 0 and  $\sqrt{a}$ . What is the value of a?

- a. 1
- b. 2
- c. 4
- d. 8

**Correct Answer:** Option b

**LOB:** Extend the law of exponents in order to simplify a given expression

1. Which of these is equivalent to  $9^{\frac{3}{4}} \times 27^{\frac{1}{5}}$  ?

- a.  $3^{\left(\frac{3}{4}+\frac{3}{5}\right)}$
- b.  $3^{\left(\frac{3}{2}+\frac{3}{5}\right)}$
- c.  $3^{\left(\frac{3}{4}-\frac{3}{5}\right)}$
- d.  $3^{\left(\frac{3}{2}-\frac{3}{5}\right)}$

**Correct Answer:** Option b

2. If  $\left(p^{\frac{1}{3}} \times 5^{\frac{1}{4}}\right)^{\frac{1}{2}} \times \left(\frac{3^4}{q^{\frac{1}{5}}}\right)^2 = \frac{3^6}{5^{40}}$ , then what are the values of p and q?

- a. p=3 and q=5
- b. p=9 and q=5
- c. p=3 and q=25
- d. p=9 and q=25

**Correct Answer:** Option d

# Suggested Teacher Resources



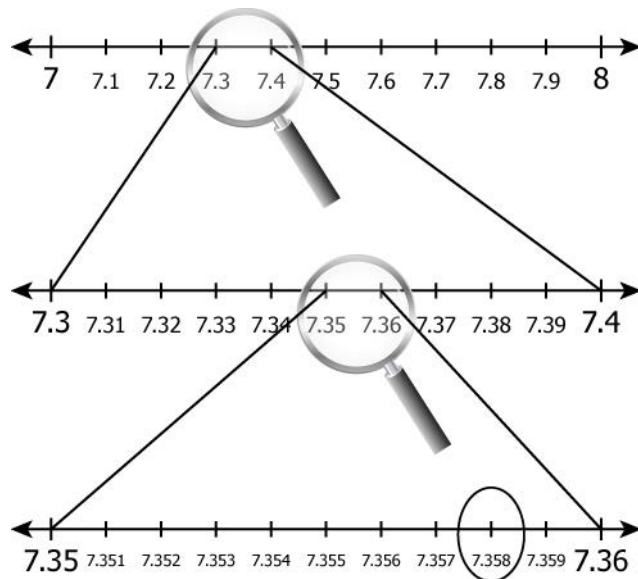
## Lesson Plan



<b>Objective</b>	Use successive magnification in order to visualise a given decimal number on a number line
<b>Prerequisite</b>	Representation of integers, rational numbers on a number line, decimal numbers, finding midpoint of a line segment
<b>Materials required</b>	Pen, notebook, ruler, meter scale, chart papers
<b>Procedure</b>	<p>Teacher will introduce the topic with the help of an activity.</p> <p><b>Activity:</b>  <b>Setup:</b> The class will be divided into 4 groups.            Each group will stand alongside one of the four walls of the classroom</p> <ol style="list-style-type: none"> <li>1. The teacher will ask each group to find the midpoint of the wall length and mark it on the wall.</li> <li>2. The teacher will give a meter scale to each group and give a bunch of chart papers for them to loosely paste it on the wall for the activity and the measurements, so that the walls remain clean.</li> <li>3. After finding its midpoint the teacher will then ask the students to take the midpoint as an extreme point on the left and the end point of the wall length as the right extreme point and instruct the students to repeat the same process to find the midpoint of this length.            This activity will help them to break the number into further decimal digits (any given two numbers).</li> <li>4. Repeat the above process at least twice. Let the students note down their observations in their notebook.            After the activity is over, each group can then list down the mid points they found at each step on the blackboard.</li> </ol> <p><b>Successive magnification process:</b>            Teacher will take the example of the activity where the students used a meter scale and found the midpoint of the wall length.</p> <p><b>Say:</b>            Suppose if the midpoint is between 7m and 8m, let it be 7.35m. Then the teacher will ask students to draw a number line on a paper representing 7 and 8 and dividing this interval into 10 equal parts.</p> <p>7.35 will lie somewhere between 7.3 and 7.4. So now, the students will find the midpoint of 7.3 and 7.4 on the number line. For this, they need to draw another number line below the previous one representing the interval between 7.3 and 7.4.            This way they can easily locate the point 7.35 on the number line. This process of visualizing numbers on the number line by successively dividing the intervals is known as the process of successive magnification which one can relate to the use of magnifying glass.</p>

Once it is clear to students how the numbers are magnified on the number line then the teacher will ask them to further magnify the above number line and find 7.358.

Final construction will look like this.



The teacher could then ask the students to visualize numbers like 5.467, 2.46555 etc. by working with their partners.

**2** **Activity** 

**Always True, Sometimes True or Never True**

The teacher will prepare a worksheet and share it with the students. A sample has been given below:

**ALWAYS SOMETIMES NEVER**

Statements that people make can generally be grouped into three different categories:

Statements that are **ALWAYS** true;

Statements that are **SOMETIMES** true; and

Statements that are **NEVER** true.

The statement: "A number that is divisible by 4 is also divisible by 2" is **ALWAYS** true because 2 is a factor of 4.

The statement: "A number that is divisible by 9 is also divisible by 6" is **SOMETIMES** true. For example, 36 is divisible by 9 and by 6, but 27 is divisible by 9, but not divisible by 6.

The statement: "The sum of two odd numbers is odd" is **NEVER** true because the sum of two odd numbers is always even.

There are 10 statements in the table given below.

Identify the statements which are Always true, sometimes true or never true. Support your answer with the help of an example when a statement is sometimes true or give a non-example when the statement is never true.

S. No.	Statement	Always true, Sometimes True or Never true
1	For any two rational numbers, a and b, $a - b = b - a$	
2	There are finite rational numbers between any two numbers.	
3	If we add, subtract, multiply or divide two irrationals, the result is an irrational number.	
4	Let a and b be any two rational numbers, If we divide a by b, the result is a rational number.	
5	If we multiply a rational and an irrational number, the result is an irrational number.	
6	Corresponding to each irrational number, there is a unique point on the number line.	
7	0 is an irrational number.	
8	The decimal expansion of every rational number is terminating recurring.	
9	For any two positive real numbers a and b, $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$	
10	For any three rational numbers a, b and c, $a \times (b \times c) = (a \times b) \times c$ .	

- Students will be given 10-15 minutes to attempt it with their partners.
- The teacher will then encourage the students to volunteer to discuss the solution for each part on board.

Reference: PISA- 2021 Mathematics Framework (Draft)



## 2. POLYNOMIALS

**QR Code:**



**Learning outcome and Learning Objectives:**

Content area/Concepts	Learning Objectives	Learning Outcome
Introduction to Polynomials	Recognise variables and their degree in a given algebraic expression in order to differentiate whether given expression is a polynomial in one variable or not	Identifies/Classifies polynomials among algebraic expressions in order to apply appropriate algebraic identities to factorise them
	Identify the degree of a given polynomial in order to classify an expression as zero, linear, quadratic and cubic polynomials	
Polynomials in one variable	Substitute the value of 'a' in a given expression $p(x)$ in order to find the value of polynomial at 'a' i.e. $p(a)$	
Zeroes of a Polynomial	Use given values for the variable 'x' in a polynomial $p(x)$ in order to identify if the given value is a zero of the polynomials	
Remainder Theorem	Using Remainder Theorem, calculate division of $p(x)$ by a linear polynomial ' $x - a$ ' in order to find that the remainder is $p(a)$ and verify using long division method.	
Factorisation of Polynomials	Apply factor theorem in order to determine if a linear polynomial ' $x-a$ ' is a factor of the given polynomial $P(x)$	
	Apply factor theorem in order to determine the value of an unknown constant 'k' in Polynomial $P(x)$ when a linear polynomial ' $x-a$ ' is a known factor of $P(x)$	
	Apply factor theorem in order to factorise a given polynomial	
	Factorise a given polynomial using splitting middle-term method and factor theorem in order to compare the results of the two	
Algebraic Identities	Point out to an algebraic identity that can be used in order to factorize a given expression	
	Select appropriate algebraic identities in order to evaluate the values of given expressions	

# Test items



**LOB:** Recognize variables and their degree in a given algebraic expression in order to differentiate whether given expression is a polynomial in one variable or not

1. Consider the expression  $x^{m-1} + 3$ ; where  $m$  is a constant. What is the least integer value of  $m$  for which the given expression is a polynomial in one variable?
- 0
  - 1
  - 2
  - 3

**Correct Answer:** Option c

2. Which of these is a polynomial in one variable?
- The perimeter of a square whose side length is represented by the expression  $\sqrt{x}$ .
  - The area of a square whose side length is represented by the expression  $1 + \sqrt{x}$ .
  - The area of a rectangle whose side lengths are represented by the expression  $2 + \sqrt{x}$  and  $\sqrt{x}$ .
  - The perimeter of a rectangle whose side lengths are represented by the expression  $x^2 + \sqrt{x}$  and  $5 - \sqrt{x}$ .

**Correct Answer:** Option d

**LOB:** Identify the degree of a given polynomial in order to classify an expression as zero, linear, quadratic and cubic polynomials

1. Consider the polynomials shown:  $x^3 + 2x$ ,  $4x$ ,  $2x^2 - 3x + 5$ ,  $0$ ,  $\frac{3}{2}x - \frac{1}{2}$   
Which of the following tables correctly classifies the given polynomials as zero, linear, quadratic and cubic polynomials?

a.

Zero Polynomial	Linear Polynomial	Quadratic Polynomial	Cubic Polynomial
0	$\frac{3}{2}x - \frac{1}{2}, 4x$	$2x^2 - 3x + 5$	$x^3 + 2x$

b.

Zero Polynomial	Linear Polynomial	Quadratic Polynomial	Cubic Polynomial
0	$4x$	$x^3 + 2x, \frac{3}{2}x - \frac{1}{2}$	$2x^2 - 3x + 5$

c.

Zero Polynomial	Linear Polynomial	Quadratic Polynomial	Cubic Polynomial
0, $4x$	$\frac{3}{2}x - \frac{1}{2}$	$2x^2 - 3x + 5,$	$x^3 + 2x$

d.

Zero Polynomial	Linear Polynomial	Quadratic Polynomial	Cubic Polynomial
0	$4x$	$2x^2 - 3x + 5,$ $\frac{3}{2}x - \frac{1}{2}$	$x^3 + 2x$

**Correct Answer:** Option a

2. Consider the expression  $x^{(m^2-1)} + 3x^{\frac{m}{2}}$ , where  $m$  is a constant. For what value of  $m$ , will the expression be a cubic polynomial?
- 2
  - 1
  - 1
  - 2

**Correct Answer:** Option d

**LOB:** Substitute the value of 'a' in a given expression  $p(x)$  in order to find the value of polynomial at 'a' i.e.  $p(a)$

1. Consider the polynomial in  $z$   $p(z) = z^4 - 2z^3 + 3$ . What is the value of the polynomial at  $z = -1$ ?
- 2
  - 3
  - 5
  - 6

**Correct Answer:** Option d

2. The value of the polynomial in  $x$ , is  $x^2 + kx + 5$ , where  $k$  is a constant. At  $x = 2$ , the value of the polynomial is 15. What is the value of the polynomial at  $x = 5$ ?
- 3
  - 18
  - 35
  - 45

**Correct Answer:** Option d

**LOB:** Use given values for the variable 'x' in a polynomial  $p(x)$  in order to identify if the given value is a zero of the polynomials

1. Which of these is a zero of the polynomials  $p(y) = 3y^3 - 16y - 8$ ?
- 8
  - 2
  - 0
  - 2

**Correct Answer:** Option b

2. Given that  $m + 2$ , where  $m$  is a positive integer, is a root of the polynomial  $q(x) = x^2 - mx - 6$ . Which of these is the value of  $m$ ?
- 1
  - 2
  - 3
  - 4

**Correct Answer:** Option a

**LOB:** Using Remainder Theorem calculate division of  $p(x)$  by a linear polynomial ' $x - a$ ' in order to find that the remainder is  $p(a)$  and verify using long division method.

1. The polynomial  $q(z) = z^3 - 4z + a$  when divided by the polynomial  $(z - 3)$  leaves remainder 5. What is the value of  $a$ ?
- 10
  - 3
  - 3
  - 10

**Correct Answer:** Option a

2. The polynomial  $p(x) = x^m + x$ , where  $m > 1$ , when divided by  $(x - a)$ , leaves remainder 6. Given that  $a$  is a positive integer, what is the value of  $m$ ?
- 2
  - 3

- c. 5  
d. 6

**Correct Answer:** Option a

**LOB:** Apply factor theorem in order to determine if a linear polynomial 'x-a' is a factor of the given polynomial P(x)

1. Which of these is a factor of the polynomial  $p(x) = x^3 + 4x + 5$ ?
- a.  $(x - 1)$   
b.  $(x + 1)$   
c.  $(x - 2)$   
d.  $(x + 2)$

**Correct Answer:** Option b

2. The polynomial  $(x - a)$ , where  $a > 0$ , is a factor of the polynomial  $q(x) = 4\sqrt{2}x^2 - \sqrt{2}$ . Which of these is a polynomial whose factor is  $(x - \frac{1}{a})$ ?
- a.  $x^2 + x + 6$   
b.  $x^2 - 5x + 4$   
c.  $x^2 + 4x - 3$   
d.  $x^2 + x - 6$

**Correct Answer:** Option d

**LOB:** Apply factor theorem in order to determine the value of an unknown constant 'k' in Polynomial P(x) when a linear polynomial x-a is a known factor of P(x)

1. The polynomial  $(x - a)$  is a factor of the polynomial  $x^4 - 2x^2 + kx + k$ , where  $k$  is a constant. Which of these is the correct relation between  $a$  and  $k$ ?
- a.  $k = \frac{a^2(2-a^2)}{1+a}$   
b.  $k = \frac{a^2(2+a^2)}{1+a}$   
c.  $k = \frac{a^2(2+a^2)}{1-a}$   
d.  $k = \frac{a^2(2-a^2)}{1-a}$

**Correct Answer:** Option a

2. The polynomial  $(4x - 3)$  is a factor of the polynomial  $q(x) = 4x^3 + x^2 - 11x + 2r$ . What is the value of  $r$ ?
- a. 2  
b. 3  
c. 4  
d. 11

**Correct Answer:** Option b

**LOB:** Apply factor theorem in order to factorize a given polynomial

1. The polynomial  $p(x) = x^3 - 5x^2 - x + 5$  is such that  $p(1) = 0$  and  $p(-1) = 0$ . Which of these is equivalent to  $p(x)$ ?
- a.  $(x - 1)(x + 5)$   
b.  $(x - 1)(x + 1)(x + 5)$   
c.  $(x - 1)(x + 1)(x - 5)$   
d.  $(x + 1)(x - 5)$

**Correct Answer:** Option c

2. A polynomial  $p(x)$  of degree  $n$  is such that  $p(a) = 0$  and  $p(-b) = 0$ . Which of the following is the factored form of the polynomial?
- a.  $(x - a)(x + b)g(x)$ ; where  $g(x)$  is a polynomial of degree  $n - 2$   
b.  $(x - a)(x + b)g(x)$ ; where  $g(x)$  is a polynomial of degree  $n$   
c.  $(x + a)(x + b)g(x)$ ; where  $g(x)$  is a polynomial of degree  $n - 2$   
d.  $(x + a)(x + b)g(x)$ ; where  $g(x)$  is a polynomial of degree  $n$

**Correct Answer:** Option a

**LOB:** Factorize a given polynomial using splitting middle-term method and factor theorem in order to compare the results of the two

1. Which of these is obtained by factorizing the polynomial  $10x^2 - 9x + 2$ ?
  - a.  $(2x - 1)(5x - 2)$
  - b.  $(2x - 1)(5x + 2)$
  - c.  $(2x + 1)(5x + 2)$
  - d.  $(2x + 1)(5x - 2)$

**Correct Answer:** Option a

2. The zeroes of the polynomial  $p(x) = x^2 - (2k + 1)x + 16$  are positive integers. Given that  $k$  is an integer, which of these is equivalent to the polynomial?
  - a.  $(x - 1)(x + 16)$
  - b.  $(x - 1)(x - 16)$
  - c.  $(x - 2)(x - 8)$
  - d.  $(x - 4)(x - 4)$

**Correct Answer:** Option b

**LOB:** Point out to an algebraic identity that can be used in order to factorize a given expression

1. Which of these identities can be used to factorize the expression  $4x^2 - 19x + 16$ ?
  - a.  $(x - a)^2 = x^2 - 2a + a^2$
  - b.  $(x + a)^2 = x^2 + 2a + a^2$
  - c.  $(x - a)(x - b) = x^2 - (a + b)x + ab$
  - d.  $(x - a)(x + a) = x^2 - a^2$

**Correct Answer:** Option c

2. The volume of a cube is given by the expression  $27x^3 + 8y^3 + 54x^2y + 36xy^2$ . What is the expression for the side length of the cube?
  - a.  $3x + 2y$
  - b.  $3x - 2y$
  - c.  $9x - 8y$
  - d.  $9x + 8y$

**Correct Answer:** Option a

**LOB:** Select appropriate algebraic identities in order to evaluate the values of given expressions

1. Which of these identities can be used to find the value of the expression  $97 \times 103$ ?
  - a.  $(x - y)^2 = x^2 - 2y + y^2$
  - b.  $(x + y)^2 = x^2 + 2y + y^2$
  - c.  $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz$
  - d.  $(x - y)(x + y) = x^2 - y^2$

**Correct Answer:** Option d

2. Given that  $100^2 = a^2$ , which expression gives the value of the expression  $103 \times 108$ ?
  - a.  $a^2 + 11a + 24$
  - b.  $a^2 + 24a + 11$
  - c.  $a^2 + 24a + 24$
  - d.  $a^2 + 11a + 11$

**Correct Answer:** Option a

# Suggested Teacher Resources



## Lesson Plan



<b>Objective</b>	Using Remainder Theorem, calculate division of polynomial $p(x)$ by a linear polynomial $(x - a)$ in order to find that the remainder is $p(a)$ and verify using long division method.
<b>Prerequisite</b>	Addition, subtraction, multiplication and division of polynomials, Evaluation of an algebraic expression for the given values of variable.
<b>Vocabulary words</b>	Factors, polynomials, remainder, degree of polynomial, power of polynomial, coefficients, variables, dividend, remainder, quotient, divisor
<b>Materials required</b>	Different coloured chalks to mark dividend, divisor, quotient, remainder
<b>Procedure</b>	<p>The teacher will begin by writing this on the board:            Dividend = ( _____ <math>\times</math> Quotient) + _____            The students have to fill in the blanks.            In the discussion that follows, the equation is to be completed like below.            Dividend = (<u>Divisor</u> <math>\times</math> Quotient) + <u>Remainder</u></p> <p>For instance, <math>48 = 5 \times 9 + 3</math></p> <p>When 48 is divided by 5, Dividend=48, Divisor= 5, Quotient=9 and Remainder=3.</p> <p>The teacher will then write the below equation involving polynomials on the board.</p> $x^4 - 1 = (x - 1) \times (x^3 + x^2 + x + 1) + 2$ <p>The teacher will ask the students to compare the equation with the Euclid's division lemma and find any similarity.            She will then ask the students to identify dividend, divisor, quotient and remainder in the equation. They will be as follows:            Dividend= <math>x^4 - 1</math>            Divisor= <math>x - 1</math>            Quotient= <math>x^3 + x^2 + x + 1</math>            Remainder= 2</p> <p>The teacher will then ask the students to divide the polynomial <math>(x^4 - 1)</math> by polynomial <math>(x - 1)</math> to verify if the quotient and the remainder come out to be the same as deduced above.            The teacher will give sometime to the students to do the division by using the long division method.</p>

$$\begin{array}{r}
 x-1)x^4 + 1(x^3 + x^2 + x + 1) \\
 \underline{-x^4 - x^3} \\
 x^3 + 1 \\
 \underline{-x^3 - x^2} \\
 x^2 + 1 \\
 \underline{-x^2 - x} \\
 x + 1 \\
 \underline{-x - 1} \\
 2
 \end{array}$$

It will be noted that,

Quotient=  $x^3 + x^2 + x + 1$  and Remainder= 2.

The teacher then repeats the whole activity by using another equation, for instance:

$$y^3 - 3y^2 + 4y + 50 = (y - 3) \times (y^2 + 4) + 62$$

The teacher then directs the attention of the students to the nature of the polynomials used in both the cases.

The divisors  $(x - 1)$  and  $(y - 3)$  are both linear polynomials.

The dividends  $(x^4 - 1)$  and  $(y^3 - 3y^2 + 4y + 50)$  are both polynomials of degree greater than 1.

If the dividend  $p(x) = x^4 - 1$  is evaluated for  $x = 1$ , then  $p(1) = 2$ .

If the dividend  $g(y) = y^3 - 3y^2 + 4y + 50$  is evaluated for  $y = 3$ , then  $g(3) = 62$ .

It can be seen that the values of  $p(1)$  and  $g(3)$  are equal to the remainders that were found during their respective long division.

The teacher will then mention that this will happen in each case where a polynomial of degree equal to or greater than one is being divided by a linear polynomial.

This is a theorem, known as the Remainder theorem, which states that- "Let  $p(x)$  be any polynomial of degree greater than or equal to one and let  $a$  be any real number. If  $p(x)$  is divided by the linear polynomial  $(x - a)$ , then the remainder is  $p(a)$ ."

Remainder theorem can be used to find the remainder without actually doing the division. This could easily be verified by the two cases considered above.

The teacher will now give 2-3 questions related to Remainder theorem to the students.

A sample has been given below:

By Remainder Theorem, find the remainder when the polynomial  $p(x) = 4x^3 - 12x^2 + 14x - 3$  is divided by polynomial  $g(x) = 2x - 1$ .

The teacher will give some time to the students to attempt the questions and then discuss them.

2

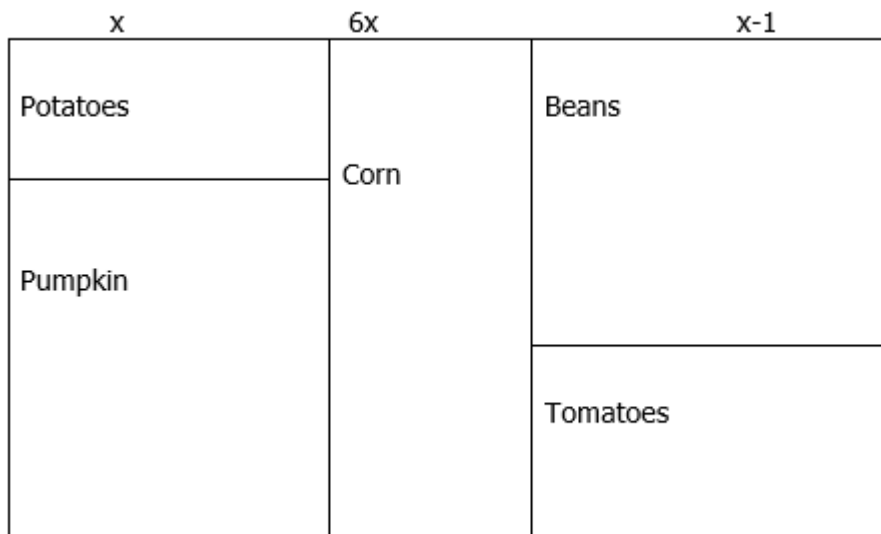
## Activity



Share the problem given below with the students either by writing it on the blackboard or by distributing printed sheets.

A farmer Ram is planting a garden this spring. He wants to plant potatoes, pumpkins, corn, beans, and tomatoes. His plan for the field layout in feet is shown in the figure below.

Use the figure and your knowledge of polynomials, perimeter, and area to solve the following:



If the area of potato field is  $4x^2$ , area of Pumpkin field is  $5x^2 + x$ , area of Corn field is  $36x^2 + 6x$ , area of bean field is  $x^4 + x^3 - 2x^2 + x - 1$  and area of tomato field is  $x^3 - 3x^2 + 3x - 1$ . Then,

1. Find the other side of the fields.
2. Find the perimeter of the pumpkin field.
3. If Ram wants to add a cucumber field near tomatoes with sides half of the pumpkin field. Find the area of the cucumber field.
4. If the value of  $x = 3$  then find the total area of the field.

**Instructions:**

- Give the students some time to attempt these problems with their partners.
- After the time is over, the teacher could encourage students to volunteer to solve the different parts one by one on the board.
- The teacher would be there to facilitate the discussion and help solve queries, if required.
- Note: A variety of problems like above could be discussed with the students in a similar manner.



# 3. COORDINATE GEOMETRY

**QR Code:**



**Learning outcome and Learning Objectives:**

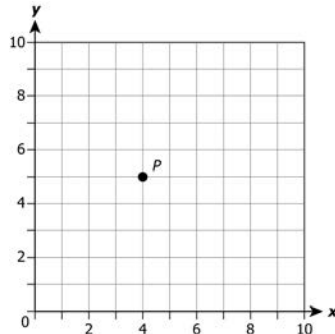
Content area/Concepts	Learning Objectives	Learning Outcome
Introduction to Coordinate Geometry	Determine the x & y co-ordinate of a point from a graph in order to write the co-ordinates of the point as an ordered pair	Develops strategies from understanding of coordinate geometry in order to locate points in a Cartesian plane
Cartesian System	Plot a point on the Cartesian plane in order to determine QUADRANT of the point	
Plotting a Point in the Plane if its Coordinates are given	Observe a given ordered pair in order to comment on its location	
Application of Coordinate Geometry	Apply concepts of coordinate geometry in order to simplify given word problems	

# Test items



**LOB:** Determine the x & y co-ordinate of a point from a graph in order to write the co-ordinates of the point as an ordered pair

1. What is the coordinate of the point P shown on the coordinate grid?

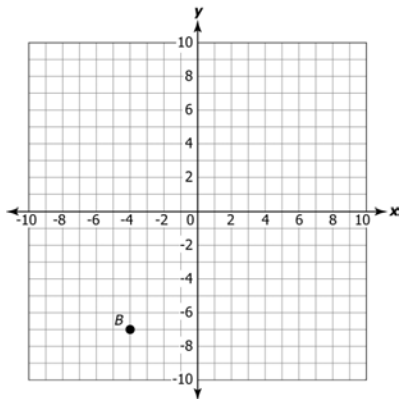


- a. (4, 5)
- b. (-4, 5)
- c. (5, 4)
- d. (5, -4)

**Correct Answer:** Option a

2. The perpendicular distance of the point A ( $m, 2n$ ) from the x-axis is 6 units. Given that  $m < 0$  and  $n > 0$ , which of these represents the point B with coordinates  $(n + 1, m)$ ?

- a.
- b.
- c.



d.

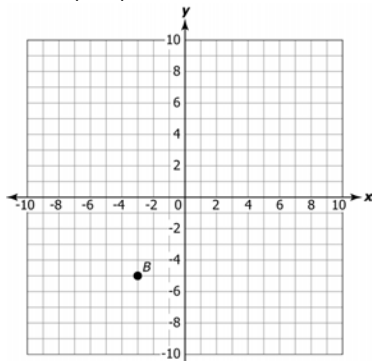
**Correct Answer:** Option c

**LOB:** Plot a point on the Cartesian plane in order to determine QUADRANT of the point

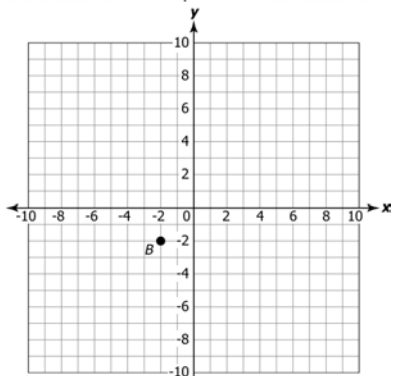
1. A point P ( $a, b$ ) is such that:  $a < 0, b > 0$ . In which quadrant does the point P lie?
  - a. First Quadrant
  - b. Second Quadrant
  - c. Third Quadrant
  - d. Fourth Quadrant

**Correct Answer:** Option b

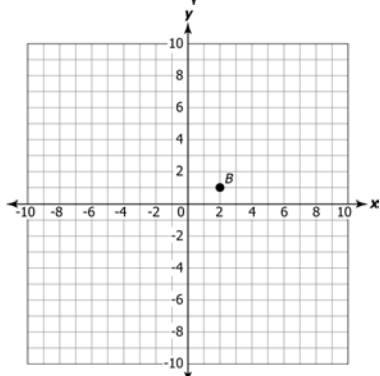
2. The point R ( $2a + 3, 2b + 1$ ) lies in the third quadrant, where  $a \neq b$ . Which of these could be the point B with coordinates ( $a, b$ )?



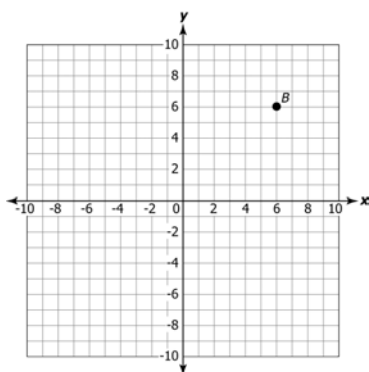
a.



b.



c.



d.

**Correct Answer:** Option a

**LOB:** Observe a given ordered pair in order to comment on its location

1. In which quadrant does the point  $(5, -7)$  lie?
  - a. First Quadrant
  - b. Second Quadrant
  - c. Third Quadrant
  - d. Fourth Quadrant

**Correct Answer:** Option d

2. The point A  $(k, k - 2)$  lies in the first quadrant and the point does not lie on any of the axis. Another point M  $(m, 2m - 5)$  is such that  $m$  is equal to the least possible integer value of  $k$ . Which of these statements is true?
  - a. Point M lies in the first quadrant.
  - b. Point M lies in the second quadrant.
  - c. Point M lies in the third quadrant.
  - d. Point M lies in the fourth quadrant.

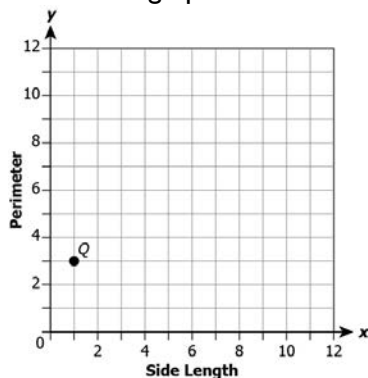
**Correct Answer:** Option a

**LOB:** Apply concepts of coordinate geometry in order to simplify given word problems

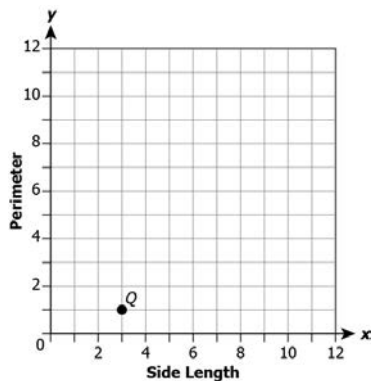
1. Amit's school is 5 km to the west and 3 km north of his house. He represented his house and his school on a coordinate grid, with his house located at the origin, and the positive x axis represent the direction that is east of his house. If 1 unit on the coordinated grid represents 1 km, what will be the coordinate of his school?
  - a.  $(5,3)$
  - b.  $(3,5)$
  - c.  $(-5,3)$
  - d.  $(-3,5)$

**Correct Answer:** Option c

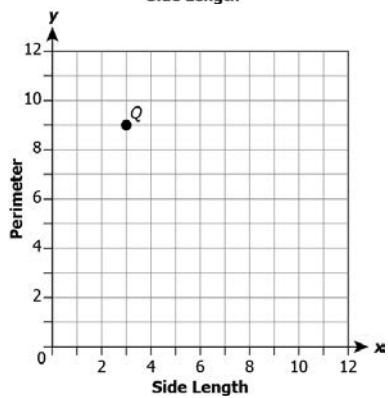
2. Riya created a graph which represents the perimeter of squares for different side lengths. Which of these could be a point on the graph that she created?



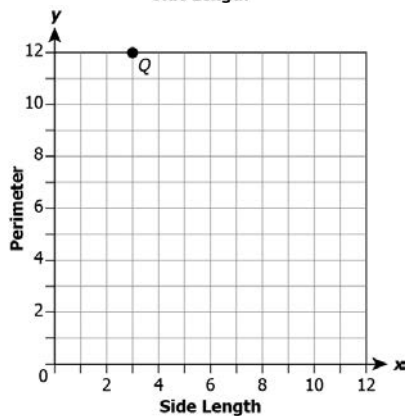
a.



b.



c.



d.

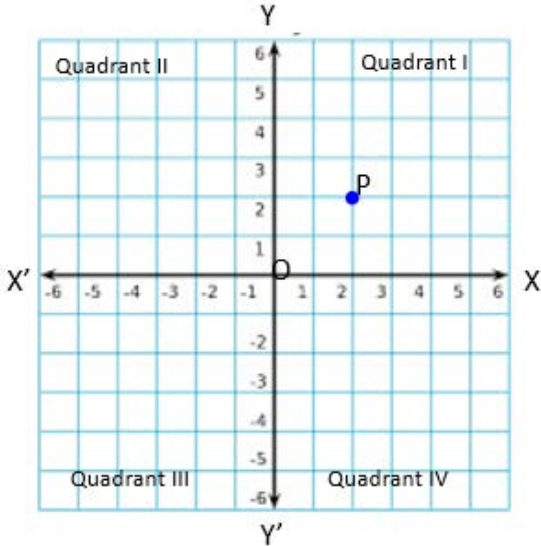
**Correct Answer:** Option d

# Suggested Teacher Resources



## Lesson Plan



<b>Objectives</b>	Determine the x & y coordinate of a point from a graph in order to write the coordinates of the point as an ordered pair
<b>Pre-requisites</b>	Basic knowledge of maps and coordinates
<b>Materials required</b>	Map (with grid lines), Rulers, Markers, Graph papers, Sheets.
<b>Procedure</b>	<ul style="list-style-type: none"> <li>• Show the class a map that includes a grid (for example, roadmap, map in an atlas, etc.). Ask students to discuss why the map would have a grid on it.</li> <li>• Tell students that just like a grid helps us find a location on a map, the Cartesian coordinate system can help us find the location of points and points on a line.</li> <li>• Divide the class into pairs, and have students create a model of the Cartesian coordinate system.</li> <li>• Provide each pair with a sheet, rulers, and markers.</li> <li>• Guide the pairs in creating the Cartesian coordinate system by drawing on the board a horizontal line <math>X'X</math> and a vertical line <math>Y'Y</math> in such a way that the two lines are perpendicular to each other.</li> <li>• The horizontal line <math>X'X</math> is called the x-axis and the vertical line <math>YY'</math> is called the y-axis. The point where both the lines intersect each other is called the 'origin' which is often denoted by 'O'.</li> <li>• Have students number each axis appropriately. Since positive numbers lie on <math>OX</math> and <math>OY</math>, <math>OX</math> and <math>OY</math> are called the positive directions of x-axis and y-axis. Similarly, <math>OX'</math> and <math>OY'</math> are the negative directions of the x- axis and the y-axis. Students can also create the other horizontal and vertical grid lines if desired.</li> <li>• Now, give the students a graph paper and ask them to replicate the cartesian plane on it as well. Discuss the difference between using a sheet and a graph paper for coordinate geometry. Emphasize on the need of a scaling paper. On a plain sheet, the markings wouldn't be regular and equidistant as compared to the one on a graph paper.</li> </ul> <div style="text-align: center;">  </div> <ul style="list-style-type: none"> <li>• It could be clearly seen that the two axes divide the cartesian plane into four equal parts</li> </ul>

which are known as 'quadrants' (meaning one fourth part). The quadrants have been numbered in the anticlockwise direction starting from OX.

- Now discuss the important terms related to the topic. This plane consisting of the two axes and the quadrants is called a cartesian plane, a coordinate plane or an xy-plane. The axes are the coordinate axes.
- Plot a point say P anywhere on the cartesian plane drawn on the graph paper, ask the students to identify its position with respect to the origin. Then discuss that the position of a point is defined by defining its coordinates. For which we first define the x-coordinate also known as the abscissa and then the y-coordinate also known as the ordinate which are placed in brackets. For instance, the coordinates of point P (2,2).
- Now, ask students to work in pairs where one of the students gives the name to a point on the graph paper and the other student has to tell its coordinates.
- Now the teacher should discuss how each quadrant is similar and different?
  - First and second quadrant both have positive y axes but have different x axes.
  - Third and Fourth quadrant both have negative y axis but different x axis.
  - First and fourth quadrant both have positive x axis but different y axis.
  - Third and Second quadrant both have negative x axis and different y axis

The teacher could then plot a few points (in all the 4 quadrants) and ask the students to work in pairs to find the coordinates of these points.

Source

<https://study.com/academy/lesson/the-cartesian-coordinate-system-lesson-plan.html>

2

## Activity



### **Activity I: Get to the point!**

The teacher will start with a story:

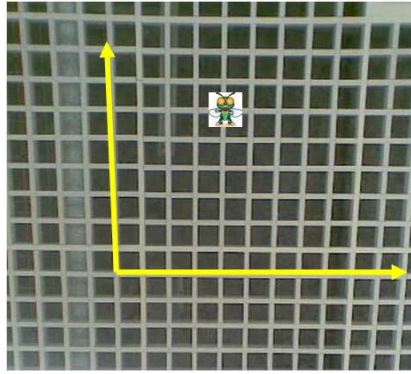
A very famous mathematician called Rene Descartes lay in bed one night. As he lay there, he looked up at the ceiling in his bedroom. He noticed a fly was asleep on the ceiling. Descartes, being a mathematician wondered if he could figure out a way of stating where exactly the fly was on the ceiling. Obviously, it has to be a precise description he thought. I can't really say, "To the left" or "Near the right" or "In the middle".

What Descartes saw could be seen in the image shared below:



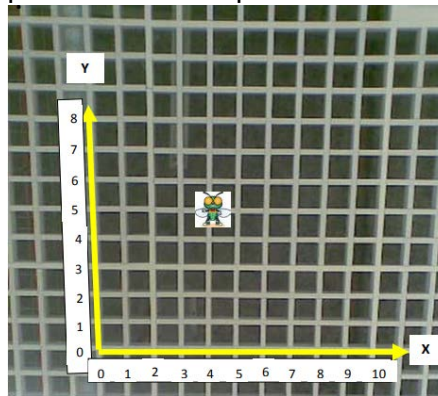
He began to think about how he might be able to describe the exact position of the fly.

Descartes decided that if he drew two lines perpendicular to each other, then he might be able to come up with a way of describing the exact position of the fly.



**How do you think this would have helped him?**

Descartes decided to place numbers on the horizontal line and on the vertical line. He could now state accurately where exactly the fly was on the ceiling. But there was a problem, should he give the vertical number of tiles followed by horizontal? i.e. go up 5 squares and move across 4 squares, or should he give the horizontal number first, then the vertical? i.e. go across 4 squares then move up 5?

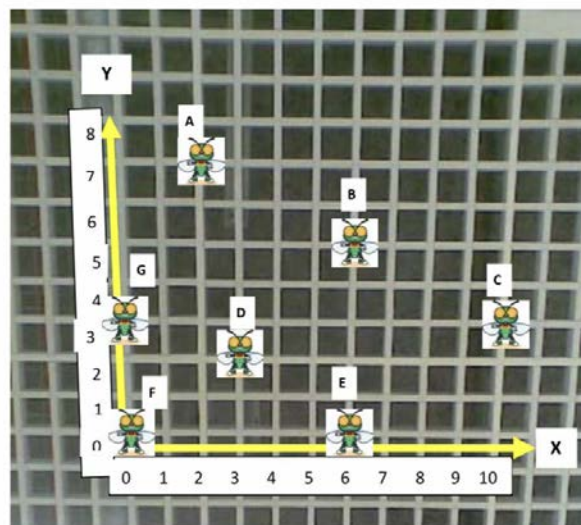


He decided to give the HORIZONTAL NUMBER FIRST and THE VERTICAL NUMBER SECOND. To help people remember this he called the horizontal line X and the vertical line Y (Because X comes before Y in the alphabet) So, in this diagram, the position of the fly can be found by moving 4 units across, then 5 units up. These are known as X, Y values and are written like this

Position of fly = (4, 5)

In honour of Rene Descartes, the graph showing the coordinates of the fly is known as the Cartesian Plane (or X Y Plane).

Now, the teacher could show the image shared below where more flies could be seen. Look at where each fly is on the coordinate plane and state where each one is, using Descartes coordinates.



Reference: <https://www.projectmaths.ie/documents/T&L/IntroductionToTheCartesianPlane.pdf?strand>



# 4. LINEAR EQUATIONS IN TWO VARIABLES

**QR Code:**



**Learning outcome and Learning Objectives:**

Content area/Concepts	Learning Objectives	Learning Outcome
Introductions	Recall concepts of coefficients and variables in order to construct a linear equation from a given statement	Relates the algebraic and graphical representations of a linear equation in one/two variables in order to apply the concepts to daily life situations
Linear Equations	Compare a given linear equation to the standard form $ax + by + c + 0$ in order to deduce the values of a, b and c	
Solutions of Linear Equations	Use substitution method in order to deduce whether the ordered pair is solution to a given linear equation	
Graph of Linear Equations	Plot the points on a graph in order to represent a linear equation in two variables	
	Solve an equation in order to represent it on a number line and a Cartesian plane	
Applications of Linear Equations	Apply principles of linear equations in order to formulate and solve for a variety of problems in real life situations	

# Test items



**LOB:** Recall concepts of coefficients and variables in order to construct a linear equation from a given statement

1. In an exhibition, the cost of tickets for an adult is ₹5 more than thrice the cost of ticket for a child. Which equation relates the cost,  $y$ , of adult ticket in terms of the cost,  $x$ , of child ticket?
  - a.  $y = 5 + 3x$
  - b.  $y + 5 = 3x$
  - c.  $y = 3 + 5x$
  - d.  $y + 3 = 5x$

**Correct Answer:** Option a

2. Ravi is a salesperson and earns a fixed salary per month plus a commission on his monthly sales. Ravi's monthly earnings is given by the equation  $y = 10,000 + 0.05x$ . Ravi gets a hike of 10% on his fixed salary and will now be earning ₹140 on every 2000 rupees worth of sales. Which equation shows Ravi's monthly earnings after the hike?
  - a.  $y = 11000 + 0.14x$
  - b.  $y = 11000 + 0.07x$
  - c.  $y = 12000 + 0.10x$
  - d.  $y = 12000 + 0.15x$

**Correct Answer:** Option b

**LOB:** Compare a given linear equation to the standard form  $ax + by + c = 0$  in order to deduce the values of  $a$ ,  $b$  and  $c$

1. Which option shows  $5y - 8x = 7(x + y) - 9$  expressed in the form of  $ax + by + c = 0$ ?
  - a.  $-x + 6y - 9 = 0$
  - b.  $-x + 12y - 9 = 0$
  - c.  $15x + 2y - 9 = 0$
  - d.  $15x - 4y - 9 = 0$

**Correct Answer:** Option c

2. In the equation shown,  $k > 0$   $-(k + 1)x + ky - 5k = 1 - 2ky$   
The equation when expressed in the form  $ax + by + c = 0$  gives  $c = 6$ . What are the values of  $a$  and  $b$ ?
  - a.  $a = 2$  and  $b = -3$
  - b.  $a = -2$  and  $b = 3$
  - c.  $a = -2$  and  $b = -1$
  - d.  $a = 2$  and  $b = 1$

**Correct Answer:** Option a

**LOB:** Use substitution method in order to deduce whether the ordered pair is solution to a given linear equation

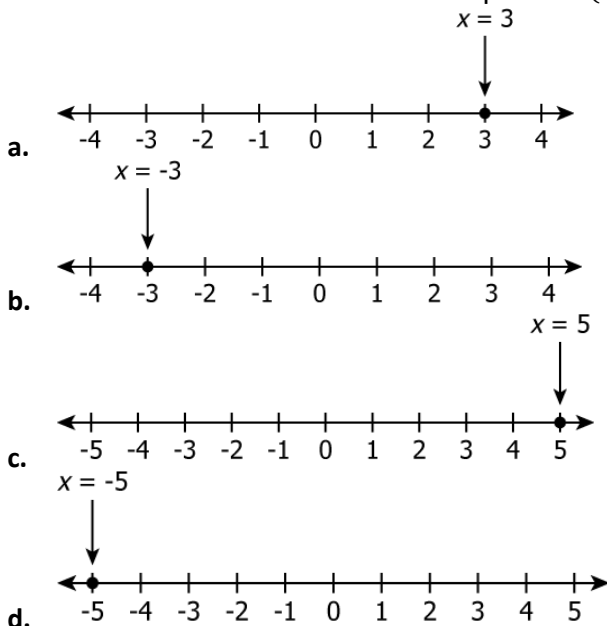
1. Which of these equations has  $(1.5, 4)$  as one of the solutions?
  - a.  $20x + 5y = 87.5$
  - b.  $20x + 5y = 50$
  - c.  $20x + 5y = 520$
  - d.  $20x + 5y = 270$
2. Rahul claims that  $(4, 3)$  is one of the solutions of the equation  $2x - y + 3 = 5$ . Is Rahul's claim correct?
  - a. Yes, as  $2(4 - 3) + 3$  simplifies to 5 and 5 is also the number on the right-hand side of the equation.
  - b. Yes, as  $2(3) - 4 + 3$  simplifies to 5 and 5 is also the number on the right-hand side of the equation.
  - c. No, but replacing 5 in the given equation with 8 will result in the equation having  $(4, 3)$  as one of its solutions.

- d. No, but replacing 5 in the given equation with 3 will result in the equation having (4, 3) as one of its solutions.

**Correct Answer:** Option c

**LOB:** Solve an equation in order to represent it on a number line and a Cartesian plane

1. Which number line shows the solution of the equation  $3(x - 3) + 2(x + 1) = 8$ ?



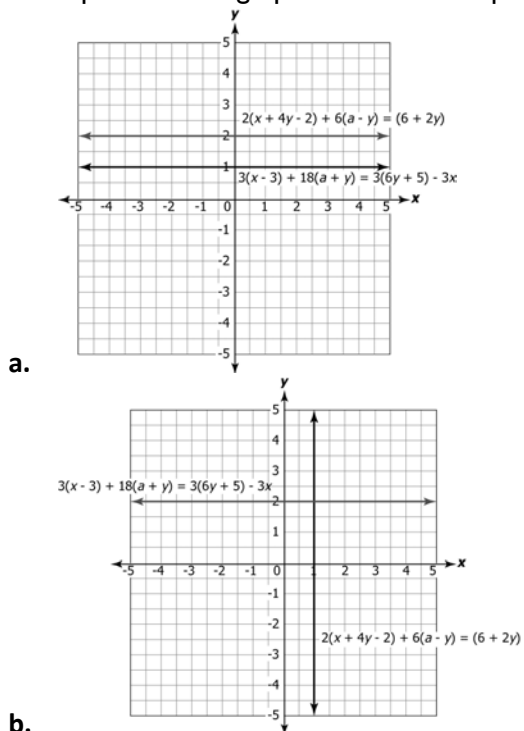
**Correct Answer:** Option a

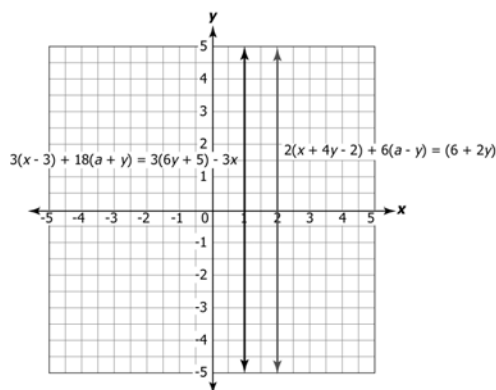
2. Consider the equations.

Equation 1:  $2(x + 4y - 2) + 6(a - y) = (6 + 2y)$

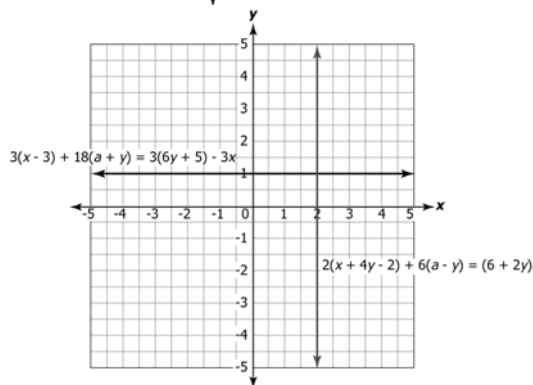
Equation 2:  $3(x - 3) + 18(a + y) = 3(6y + 5) - 3x$

Which of these represents the graph of the above equations?





c.



d.

**Correct Answer:** Option c

**LOB:** Apply principles of linear equations in order to formulate and solve for a variety of problems in real life situations

1. A person invested money in an account and earns interest of ₹100 every month. Which equation shows the total interest,  $y$ , the person will earn after  $x$  months and how much interest will the person earn in 1 year?
  - a.  $y = 12x$ ; 144
  - b.  $y = 12x + 100$ ; 154
  - c.  $y = 100x$ ; 120
  - d.  $y = 100x$ ; 1200

**Correct Answer:** Option d

2. A swimming academy charges onetime registration fees plus fixed monthly fee. Mayank joined the academy and paid ₹1200 in the first month including registration fee and ₹700 in the second month. Which linear equation shows the total fee,  $y$ , for  $x$  months and the total fee for 6 months?
  - a.  $y = 1200 + 700x$ ; 5400
  - b.  $y = 1200 + 700x$ ; 4700
  - c.  $y = 500 + 700x$ ; 4700
  - d.  $y = 500 + 700x$ ; 5400

**Correct Answer:** Option c

# Suggested Teacher Resources



## Lesson Plan



<b>Objective</b>	Use substitution method in order to find solutions for a given linear equation in two variables														
<b>Prior knowledge</b>	Linear equations in two variables, its standard form														
<b>Materials required</b>	Graph paper, ruler, pencil.														
<b>Procedure</b>	<p>The teacher will start the class with a question.</p> <p>Rohan tells you that he got 32 points in a basketball game. Write down all the possible ways he could have scored 32 with only two- and three-point baskets. Use the table below to organize your work.</p> <table border="1" data-bbox="603 913 1185 1256"> <thead> <tr> <th>Number of Two-Pointers</th> <th>Number of Three-Pointers</th> </tr> </thead> <tbody> <tr> <td>16</td> <td>0</td> </tr> <tr> <td>13</td> <td>2</td> </tr> <tr> <td>10</td> <td>4</td> </tr> <tr> <td>7</td> <td>6</td> </tr> <tr> <td>4</td> <td>8</td> </tr> <tr> <td>1</td> <td>10</td> </tr> </tbody> </table> <p>The teacher will give some time to the students to read the question and then discuss as below:</p> <p>We can write an equation to represent Rohan's score at the basketball game. Let <math>x</math> be the number of two-pointers and <math>y</math> be the number of three-pointers that Rohan scored. as</p> $2x + 3y = 32$ <p>This is an example of a linear equation in two variables. An equation in the form of <math>ax + by = c</math> is called a <i>linear equation in two variables</i>, where <math>a</math>, <math>b</math>, and <math>c</math> are constants, and at least one of <math>a</math> and <math>b</math> are not zero.</p> <p>The equation is in the standard form. An equation of this form, <math>ax + by = c</math>, is referred to as an equation in <i>standard form</i>.</p> <p>The teacher will then ask the students- What pairs of numbers did you find that worked for Rohan's basketball score? Did just any pair of numbers work?</p> <p>No, not just any pair of numbers worked. Students should identify the pairs of numbers in the table given in the question. For example, we couldn't say that Rohan scored 15 two-pointers and 1 three-pointer because that would mean she scored 33 points in the game, and she only scored 32 points.</p> <p>A <i>solution</i> to the linear equation in two variables is an ordered pair of numbers <math>(x, y)</math> so that <math>x</math> and <math>y</math> makes the equation a true statement. The pairs of numbers that are given in the table for</p>	Number of Two-Pointers	Number of Three-Pointers	16	0	13	2	10	4	7	6	4	8	1	10
Number of Two-Pointers	Number of Three-Pointers														
16	0														
13	2														
10	4														
7	6														
4	8														
1	10														

Rohan are solutions to the equation  $2x + 3y = 32$  because they are pairs of numbers that make the equation true.

The teacher will then mention that a linear equation in two variables has infinitely many solutions. As in the question discussed, students could easily find infinitely many ordered pairs (solutions).

The question becomes, how do we find an unlimited number of solutions to a given linear equation?

The teacher would then explain as below:

We need to guess numbers until we find a pair that makes the equation true.

A strategy that will help us find solutions to a linear equation in two variables is as follows: We fix a number for  $x$ . That means we pick any number we want and call it  $x$ . Since we know how to solve a linear equation in one variable, then we solve for  $y$ . The number we picked for  $x$  and the number we get when we solve for  $y$  is the ordered pair  $(x, y)$ , which is a solution to the two-variable linear equation.

For example, let  $x = 5$ . Then, substitute this value in the equation  $-50x + y = 15$ , we have

$$\begin{aligned} -50(5) + y &= 15 \\ -250 + y &= 15 \\ -250 + 250 + y &= 15 + 250 \\ y &= 265. \end{aligned}$$

Therefore,  $(5, 265)$  is a solution to the equation  $-50x + y = 15$ .

Here we have used the substitution method.

Similarly, we can fix a number for  $y$  and solve for  $x$ . Let  $y = 10$ ; then

$$\begin{aligned} -50x + 10 &= 15 \\ -50x + 10 - 10 &= 15 - 10 \\ -50x &= 5 \\ \frac{-50}{-50}x &= \frac{5}{-50} \\ x &= -\frac{1}{10}. \end{aligned}$$

Therefore,  $(-\frac{1}{10}, 10)$  is a solution to the equation  $-50x + y = 15$ .

Ask students to provide a number for  $x$  or  $y$  and demonstrate how to find a solution. This can be done more than once in order to prove to students that they can find a solution no matter which number they choose to fix for  $x$  or  $y$ .

The teacher will then give a few linear equations to the students and ask them to find five solutions for each by the method discussed above. A few linear equations have been given below for the teacher's reference:

1.  $x + y = 3$
2.  $5x - 9y = 32$

Reference

<https://www.engageny.org/resource/grade-8-mathematics-module-4-topic-b-lesson-12>

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## Activity



**Materials required:** Different sets of cards with linear equations in two variables and their solution written on them.

This game is planned for use with 30 students; however, more cards can be made for play in a larger-sized class. The cards have to be prepared in advance. Each card should have a letter of the alphabet (in this case, A to O) (15 Alphabets) written on it along with a linear expression or a solution; there will be two different cards with the same letter having linear expression on one and its true or false solution on other. For example, see the list below. For the letter A there are two cards:

One card has A written on it with the linear expression  $4x + 2y = 6$   
 The other card has A written on it with the solution (1,1)

Card	Expression On Card 1	Expression On Card 2	Answer
A	$4x + 2y = 6$	(1,1)	True Solution (T)
B	$6x - 7y = 0$	(0,7)	False Solution (F)
C	$7x - 3y = 13$	(1, -2)	T
D	$6x + 16 = 2y$	(-1,5)	T

Create additional pairs of cards in the same manner up to letter O depending upon the rigor of the classroom.

**Instructions:**

If you have a class of 30 students, shuffle the set of 30 cards and distribute a card to each student. (If you have fewer or more students, shuffle a set of letter cards for each pair of students.) Allow students who get the same alphabet cards to sit together and solve the equation for the value of the variable. For example, the pair of students who got the two cards with the letter A on them will verify the solution of the linear equation:

$$4x + 2y = 6 \quad \therefore (1,1)$$

Once students have solved their equations, you might place lettered slips (in this games example, one slip with each letter A to O) in a bowl or hat. Draw out a slip and read the letter that is written on it. Invite the pair of students who have that letter on their cards to come up to the board to show how they solved their equation. If they do it correctly, they win that round of the game.

Let all student pairs who correctly solved their equations play another round of the game (with new cards or the same ones).

Variation possible in new cards –

Word problem on one card and different linear equations in two variables on another and ask to choose the correct equation.

A Linear equation in two variables on one card and other card is blank and ask the students to represent the equation on a graph.

With each repeat of the game, more pairs of students could be eliminated. Play until there is a final winner (a pair of champions).

**Source -:** Pinterest

<https://betterlesson.com/lesson/573847/graphing-linear-systems-of-equations-day-2-of-2>

# 5. INTRODUCTION TO EUCLID GEOMETRY

**QR Code:**



**Learning outcome and Learning Objectives:**

Content area/Concepts	Learning Objectives	Learning Outcome
Introduction	Give examples of theorems, postulates and axioms in order to differentiate between them with examples	Applies axiomatic approach and derives proofs of mathematical statements particularly related to geometrical concepts, like parallel lines, triangles, quadrilaterals, circles etc. in order to solve problems using them
Euclid's Definitions, Axioms and Postulates	Reproduce Euclid's axioms in your own words in order to give examples for each	
	List Euclid's 5 postulates in order to visualize and illustrate them through a diagram	
	Analyse given statements/postulates in order to determine if they are extensions of Euclid's postulates	
	Apply Euclid's postulates in order to prove basic geometrical concepts about lines, points, planes, shapes, etc	
Equivalent Versions of Euclid's Fifth Postulate	Illustrate the equivalent of Euclid's fifth postulate through a diagram in order to list conditions for two lines to be parallel	



# Test items



**LOB:** Give examples of theorems, postulates and axioms in order to differentiate between them with examples

1. Consider the statements below.

Statement 1: A straight line can be drawn joining any two points.

Statement 2: Two distinct lines can have only one point common.

Which of these is true?

- Statement 1 is a postulate and Statement 2 is a theorem.
- Statement 1 is a theorem and Statement 2 is a postulate.
- Both statements are theorems.
- Both statements are postulates.

**Correct Answer:** Option a

2. Consider the statements below.

Statement 1: Things that are equal to the same thing are equal to one another.

Statement 2: The whole is greater than the part.

Which of these is true?

- Statement 1 is a theorem and Statement 2 is an axiom.
- Statement 1 is an axiom and Statement 2 is a theorem.
- Both statements are theorems.
- Both statements are axioms.

**Correct Answer:** Option d

**LOB:** Reproduce Euclid's axioms in your own words in order to give examples for each

1. Two quantities A and B are such that  $A = B$ . Which of these equations illustrates the Euclid's axiom "If equals are added to equals, the wholes are equals"?

- $A + x = B - x$
- $A + x = B + x$
- $A + x = B$
- $A \times x = B$

**Correct Answer:** Option b

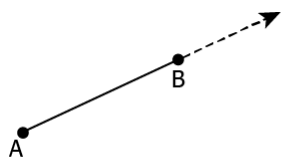
2. Two tanks have equal volume of water. After 10 L of water is taken out from each tank, the tanks still have the same volume. Which of these axioms is demonstrated by this situation?

- If equals are added to equals, the wholes are equals.
- If equals are subtracted from equals, the wholes are equals
- Things which are equal to the same thing are equal to one another.
- A whole is greater than the part.

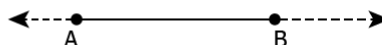
**Correct Answer:** Option b

**LOB:** List Euclid's 5 postulates in order to visualize and illustrate them through a diagram

1. To illustrate the postulate "A line segment can be produced indefinitely", two students drew the following diagrams.



**Student 1**



**Student 2**

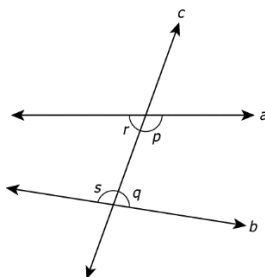
Which of these is a true statement?

- Only Student 1 is correct.
- Only Student 2 is correct.

- c. Both students are correct.
- d. Neither student is correct.

**Correct Answer:** Option c

2. Observe the figure shown.



A student claimed that the lines when extended meet at a point which lies on the left of the line c. Given that the student's claim is true, which of these justifies the claim?

- a.  $p + q < 180^\circ$
- b.  $r + s < 180^\circ$
- c.  $p + r < 180^\circ$
- d.  $s + q < 180^\circ$

**Correct Answer:** Option b

**LOB:** Analyze given statements/postulates in order to determine if they are extensions of Euclid's postulates

1. Based on Euclid's postulate, how many line(s) can pass through 3 collinear points?
  - a. 1
  - b. 2
  - c. 3
  - d. Infinite

**Correct Answer:** Option a

2. Raj drew a line passing through the points P, Q, and R. Kiran drew a line passing through Q and R. Which statement about the lines they drew is correct?
  - a. The lines coincide.
  - b. The lines are parallel.
  - c. The lines are perpendicular.
  - d. The lines meet at two points but are not perpendicular.

**Correct Answer:** Option a

**LOB:** Apply Euclid's postulates in order to prove basic geometrical concepts about lines, points, planes, shapes, etc.

1. Consider the given statement:  
From the vertex of a triangle ABC, only one median can be drawn.  
Which of the following helps prove the given statement?
  - a. A circle can be drawn with any centre and any radius.
  - b. A terminated line segment can be drawn indefinitely.
  - c. Given any two points, only one line can be drawn passing through them.
  - d. All right angles are equal to one another.

**Correct Answer:** Option c

2. Rajat drew a circle with centre M and radius 10 cm. He then made following claims using Euclid's postulates.
 

**Claim 1:** It is possible to construct infinite circles each with center M and that lie inside the given circle.

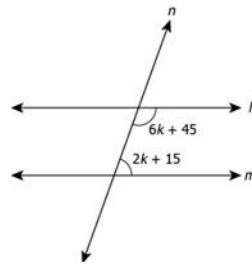
**Claim 2:** It is possible to construct infinite circles each with center M and that lie outside the given circle.

 Which statement is true?
  - a. only Claim 1 is correct
  - b. only Claim 2 is correct
  - c. both the claims are correct
  - d. neither of the claims is correct

**Correct Answer:** Option c

**LOB:** Illustrate the equivalent of Euclid's fifth postulate through a diagram in order to list conditions for two lines to be parallel

1. Observe the figure shown

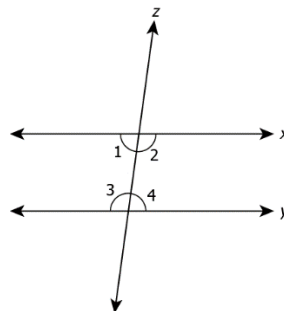


For what value of  $k$  will the lines  $l$  and  $m$  be parallel?

- a. 15
- b. 30
- c. 45
- d. 60

**Correct Answer:** Option a

2. Observe the figure shown.



Consider the given conditions for the two lines  $x$  and  $y$  to be parallel.

Condition 1:  $\angle 1 = \angle 2 = 90^\circ$

Condition 2:  $\angle 3 = \angle 4 = 90^\circ$

Which condition(s) is/are required so that the lines  $x$  and  $y$  are parallel?

- a. Condition 1 alone is sufficient.
- b. Condition 2 alone is sufficient.
- c. Both conditions are necessary.
- d. Neither condition is sufficient.

**Correct Answer:** Option c

# Suggested Teacher Resources



## Lesson Plan



<b>Objectives</b>	Students will be able to understand the intuitive need of axioms
<b>Prerequisite Knowledge</b>	Basic knowledge of terminology used in geometry such as circle, point, lines, regions etc.
<b>Material Required</b>	Chalk and blackboard, or markers and whiteboard
<b>Procedure</b>	<p>Teacher will start the class by defining a rectangle.</p> <p>She will state “ a rectangle is a quadrilateral with four right angles” and will ask students if they can simplify the definition more by defining what a quadrilateral is, a right angle is?</p> <p>A student might mention:</p> <p>Quadrilateral is a polygon made up of four-line segments.</p> <p>Polygon is a simple closed figure made up of three or more-line segments.</p> <p>Line segment is a part of a line with two endpoints.</p> <p>After stating those definitions, then again, she will ask them to define the terms used in their definitions further.</p> <p>Line: Undefined term</p> <p>Point: Undefined term</p> <p>Angle: A figure formed by two rays with a common initial point.</p> <p>Ray: Part of a line with one end point.</p> <p>Right angle: Angle whose measure is <math>90^\circ</math>.</p> <p>So, to define one thing, you need to define many other things, and you may get a long chain of definitions without an end, for example, you might get the term ‘point’ in one of the definitions which is very difficult to simplify/define further.</p> <p>For such reasons, mathematicians agreed to leave some geometric terms undefined. However, we do have an intuitive feeling for the geometric concept used. So, we represent a point as a dot, even though a dot has some dimension defined.</p> <p>Because of this, a few terms are kept undefined while developing any course of study. So, in geometry, we take a point, a line and a plane (in Euclid ‘s words a plane surface) as undefined terms. The only thing is that we can represent them intuitively, or explain them with the help of ‘physical models.</p>

For convenience, Euclid assumed certain concepts to be universal truths and left them unproven.

Then the teacher will ask the students to think about such concepts which cannot be defined/explained further.

After a thorough discussion, teacher will state that some of the Euclid's axioms, not in his order, are given below:

(1) Things which are equal to the same thing are equal to one another.

(2) If equals are added to equals, the wholes are equal.

(3) If equals are subtracted from equals, the remainders are equal.

(4) Things which coincide with one another are equal to one another.

(5) The whole is greater than the part.

(6) Things which are double of the same things are equal to one another.

(7) Things which are halves of the same things are equal to one another.

Now the teacher will discuss the above axioms by taking varied examples and introducing Postulates in the same way.

**Source**

- <http://ncert.nic.in/textbook/textbook.htm?iemh1=5-16>
- <http://ncert.nic.in/ncerts//ieep205.pdf>

2

## Worksheet



Q1. It is known that  $w + x + y = 100$  and that  $w + x = z$ . Show that  $z + y = 100$ ?

Q2. Shyam got the same number of mangoes as Ram. Rani also got the same number of mangoes as Ram. State the Euclid's axiom that illustrates the relative number of mangoes with Shyam and Rani.

Q3. Write whether the following statements are True or False? Justify your answer.

- If the area of a triangle equals the area of a rectangle and the area of the rectangle equals that of a square, then the area of the triangle also equals the area of the square.
- Let A and B be two points. There can be only two lines that could be drawn from the given two points.



A



B

c) Things which are triple the same thing are equal.

d) Whole is always bigger than the part.

Reference: <http://ncert.nic.in/ncerts//ieep205.pdf>

# 6. LINES AND ANGLES

**QR Code:**



**Learning outcome and Learning Objectives:**

Content area/Concepts	Learning Objectives	Learning Outcome
Basic Terms and Definitions	Define segment, ray, collinear points, non-collinear points, acute angle, right angle, obtuse angle, straight angle, reflex angle, complementary angles, Supplementary angles in order to identify them in a given figure	Applies axiomatic approach and derives proofs of mathematical statements particularly related to geometrical concepts, like parallel lines, triangles, quadrilaterals, circles etc. in order to solve problems using them
Pairs of Angles	Label angles created by 2 intersecting lines in order to identify vertically opposite pairs, adjacent angles, linear pairs, complementary/supplementary pairs of angles	
	Apply the concepts of linear pairs of angles and vertically opposite angles in order to establish relationships between the angles in a given figure and solve for missing values	
Parallel Lines and a Transversal	Label angles created by a transversal intersecting two parallel lines in order to identify corresponding angles, alternate angles, interior angles and define relationship between these angles	
Lines Parallel to the same Line	Solve for the value of unknown angles created by a transversal in a given figure in order to infer if the lines are parallel or not	
Angle Sum Property of a Triangle	Define relationship between angles formed when a triangle is placed between two parallel lines in order to prove that exterior angle of a triangle is the sum of the two opposite interior angles	

# Test items



**LOB:** Define segment, ray, collinear points, non-collinear points, acute angle, right angle, obtuse angle, straight angle, reflex angle, complementary angles, Supplementary angles in order to identify them in a given figure

1. Tina drew a figure and named it  $\overleftrightarrow{JK}$ . Which of the following best describes the figure Tina drew?
  - a. It is an angle.
  - b. It is a line segment.
  - c. It could be a line segment or a ray.
  - d. It could be a line or a line segment.

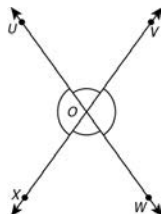
**Correct Answer:** Option c

2. Two angles are supplementary. One of them is an acute angle. Which of these could be the measure of the other angle?
  - a.  $60^\circ$
  - b.  $90^\circ$
  - c.  $120^\circ$
  - d.  $180^\circ$

**Correct Answer:** Option c

**LOB:** Label angles created by 2 intersecting lines in order to identify vertically opposite pairs, adjacent angles, linear pairs, complementary/supplementary pairs of angles

1. Two intersecting lines are shown.



Which of these statements is FALSE about the given figure?

- a.  $\angle XOW$  and  $\angle UOV$  is a pair of vertically opposite angles.
- b.  $\angle UOV$  and  $\angle VOW$  are supplementary angles.
- c.  $\angle UOX$  and  $\angle XOW$  are linear pair of angles.
- d.  $\angle UOX$  and  $\angle VOW$  are adjacent angles.

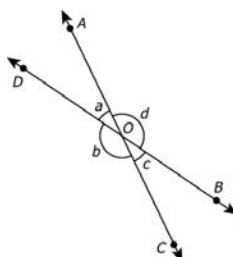
**Correct Answer:** Option d

2. Which words complete the statement below?  
When two lines intersect, \_\_\_\_\_ pairs of adjacent angles are formed and all pairs are \_\_\_\_\_.
- a. 2; supplementary
  - b. 4; supplementary
  - c. 2; complementary
  - d. 4; complementary

**Correct Answer:** Option b

**LOB:** Apply the concepts of linear pairs of angles and vertically opposite angles in order to establish relationships between the angles in a given figure and solve for missing values

1. Two intersecting lines are shown.

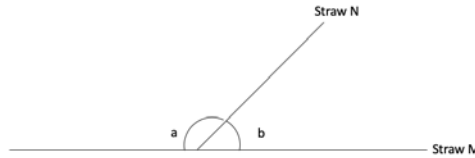


If  $d = 150^\circ$ , what are the measures of unknown angles?

- $a = 30^\circ, b = 150^\circ, c = 30^\circ$
- $a = 150^\circ, b = 150^\circ, c = 30^\circ$
- $a = 30^\circ, b = 30^\circ, c = 150^\circ$
- $a = 30^\circ, b = 150^\circ, c = 150^\circ$

**Correct Answer:** Option a

2. Harish places two straws forming angles  $a$  and  $b$  as shown.



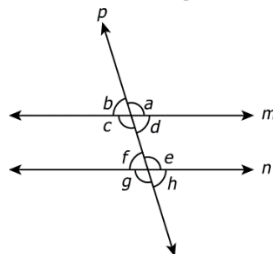
Harish moves Straw N such that the value of  $b$  triples. How does the value of  $a$  change?

- The value of  $a$  triples.
- The value of  $a$  reduces by  $2b$ .
- The value of  $a$  increases by  $2b$ .
- The value of  $a$  becomes  $\frac{1}{3}$  times.

**Correct Answer:** Option b

**LOB:** Label angles created by a transversal intersecting two parallel lines in order to identify corresponding angles, alternate angles, interior angles and define relationship between these angles

1. In the figure below,  $m$  and  $n$  are two parallel lines and  $p$  is a transversal.

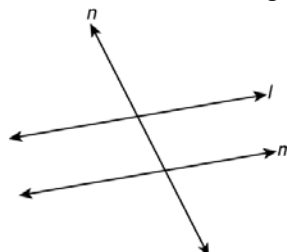


Which of these statements is true about the angles shown in the figure?

- $c + e = 180^\circ$  as  $c$  and  $e$  are alternate interior angles.
- $d = h$  as  $d$  and  $h$  are corresponding angles.
- $b = g$  as  $b$  and  $g$  are corresponding angles.
- $a = e$  as  $a$  and  $e$  are alternate interior angles.

**Correct Answer:** Option b

2. In the figure below,  $l$  and  $m$  are two parallel lines intersecting by a transversal  $n$ .



If another line is drawn parallel to the line  $m$ , what would be the increase in the pairs of alternate interior angles that will be formed?

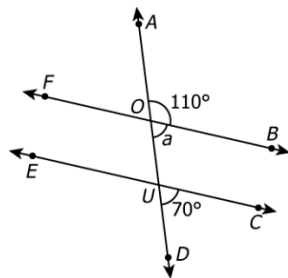
- 2
- 8
- 4
- 6

**Correct Answer:** Option c



**LOB:** Solve for the value of unknown angles created by a transversal in a given figure in order to infer If the lines are parallel or not

1. Two lines are cut by a transversal as shown.

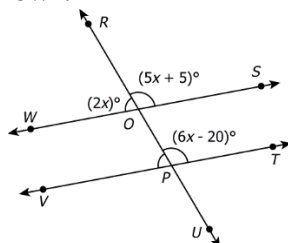


Is BF parallel to CE? Why or why not?

- Yes, because  $a = \angle CUD = 70^\circ$  and congruent alternate interior angles indicates that BF and CE are parallel.
- Yes, because  $a = \angle CUD = 70^\circ$  and congruent corresponding angles indicates that BF and CE are parallel.
- No, because  $a \neq \angle CUD$  indicating that corresponding angles are not congruent
- No, because  $\angle AOB$  and  $\angle CUD$  are alternate interior angles but not equal in measure.

**Correct Answer:** Option b

2. Two lines are cut by a transversal as shown.



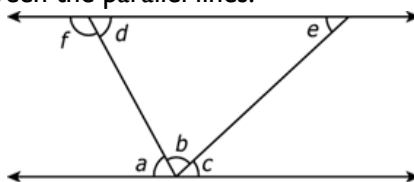
Is it true that lines SW and VT are parallel?

- Yes, because corresponding angles,  $\angle TPO$  and  $\angle SOR$  are equal in measure and their measure is  $130^\circ$ .
- No, because  $\angle TPO = 155^\circ$  and  $\angle SOR = 160^\circ$ ; this indicate that corresponding angles are not equal.
- No, because  $\angle TPO = 130^\circ$  and  $\angle SOR = 150^\circ$ ; this indicate that corresponding angles are not equal.
- Yes, because alternate angles,  $\angle TPO$  and  $\angle SOR$  are equal in measure and their measure is  $130^\circ$ .

**Correct Answer:** Option a

**LOB:** Define relationship between angles formed when a triangle is placed between two parallel lines in order to prove that exterior angle of a triangle is the sum of the two opposite interior angles

1. Consider a triangle that lies between the parallel lines.

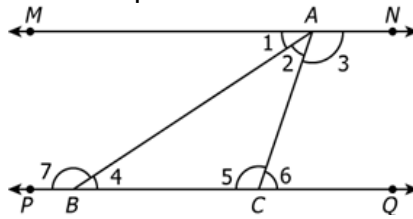


Is it true that  $\angle f = \angle b + \angle e$ ? If yes, which of these explains why?

- Yes; As  $\angle f = \angle b + \angle c$  and  $\angle c = \angle e$
- Yes; As  $\angle f = \angle b$  and  $\angle c = \angle e$
- The given statement is not true.
- Yes; As  $\angle f = \angle b$  and  $\angle e = \angle b + \angle c$

**Correct Answer:** Option a

2. In the figure below,  $\triangle ABC$  lies between the parallel lines.



Which property/postulate is used to prove  $\angle 4 + \angle 2 = \angle 6$ ?

- a. Corresponding angles are congruent
- b. Alternate interior angles are congruent
- c. Linear pair postulate
- d. Angle sum property of triangle

**Correct Answer:** Option b

# Suggested Teacher Resources



## Lesson Plan



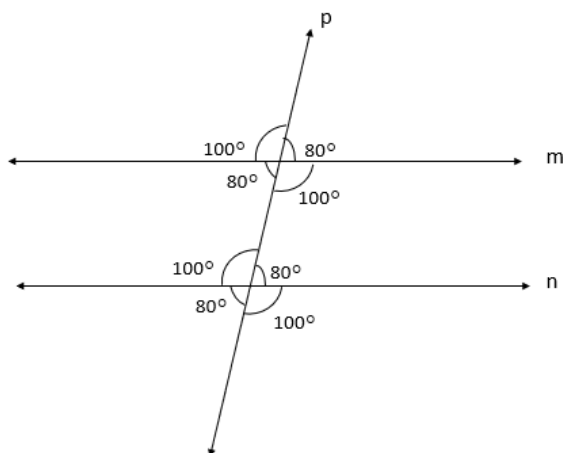
Objectives	Label angles created by a transversal intersecting two parallel lines in order to identify corresponding angles, alternate angles, interior angles and define relationship between these angles.
Prerequisites	Parallel lines, intersecting lines, transversal, corresponding angles, alternate interior angles, etc.
Material Required	Ruler, pencil, protractor.
Procedure	<p>The teacher will ask the following questions:</p> <ul style="list-style-type: none"> <li>• What is a <b>transversal</b>? A line that intersects two or more lines at different points is called a transversal. The teacher will draw two parallel lines on the board by using a ruler ensuring that these are parallel and then draw a transversal p. Similarly, the teacher will draw two non-parallel (or intersecting) lines and a transversal z</li> </ul> <p>The students also have to draw the same in their notebooks. They can consider any two lines in their lined notebooks as m and n to ensure that they indeed make parallel lines.</p> <ul style="list-style-type: none"> <li>• If we have two parallel lines or non-parallel lines cut by a transversal, how many angles are created? 8</li> </ul> <div style="text-align: center;"> </div> <ul style="list-style-type: none"> <li>• What are <b>exterior angles</b>? The angles that lie on the outside of the parallel lines (or non-parallel lines) cut by the transversal are the exterior angles. In the given figure, <math>\angle 1</math>, <math>\angle 2</math>, <math>\angle 7</math> and <math>\angle 8</math> are exterior angles.</li> <li>• What are <b>interior angles</b>? The angles that lie between the parallel lines (or non-parallel lines) cut by the transversal. <math>\angle 3</math>, <math>\angle 4</math>, <math>\angle 5</math> and <math>\angle 6</math> are interior angles.</li> <li>• What are <b>vertically opposite angles</b>?</li> </ul>

A pair of angles opposite to each other formed by two intersecting lines. For example,  $\angle 2$  and  $\angle 4$ ,  $\angle 1$  and  $\angle 3$ ,  $\angle 5$  and  $\angle 7$ ,  $\angle 6$  and  $\angle 8$  are pairs of vertically opposite angles. These are always equal. That means,  $\angle 2 = \angle 4$ ,  $\angle 1 = \angle 3$ ,  $\angle 5 = \angle 7$  and  $\angle 6 = \angle 8$ .

- What are **alternate interior angles**?  
Pair of angles between the parallel lines and on opposite sides of the transversal. In the given figure,  $\angle 3$  and  $\angle 5$ ,  $\angle 4$  and  $\angle 6$  are alternate interior angles.
- What are **alternate exterior angles**?  
Pair of angles on the outside of the parallel lines and on opposite sides of the transversal. In the given figure,  $\angle 1$  and  $\angle 7$ ,  $\angle 2$  and  $\angle 8$  are alternate exterior angles.
- What is the difference between alternate interior angles and alternate exterior angles? (Alternate interior angles are between the parallel lines and on opposite sides of the transversal while alternate exterior angles are outside the parallel lines and on opposite sides of the transversal.)
- What are **corresponding angles**?  
If two angles occupy corresponding positions, they are called corresponding angles. In the given figure,  $\angle 1$  and  $\angle 5$ ,  $\angle 2$  and  $\angle 6$ ,  $\angle 4$  and  $\angle 8$ ,  $\angle 3$  and  $\angle 7$  are pairs of corresponding angles.

The teacher will then ask the students to find the measures of all the 8 angles in the figure where a transversal intersects two non-parallel lines (intersecting lines) drawn in their notebooks using a protractor and note down their observations.

Then the teacher will ask them to measure all the angles of the figure where the transversal intersects two parallel lines. The teacher will also measure the angles in the two figures drawn on board. A sample has been given below for the parallel lines and the transversal.



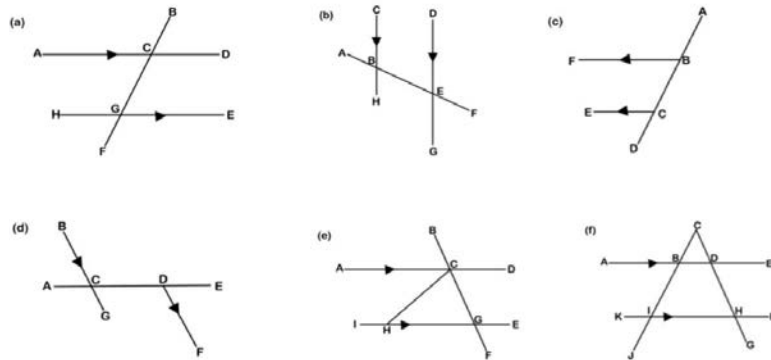
There could be a variety of different measures that the students would have found for the case **when a transversal intersects two parallel lines** the relation among the angles would remain consistent (assuming that each student drew correct parallel lines).

As is visible in the sample figure, it could easily be deduced that,

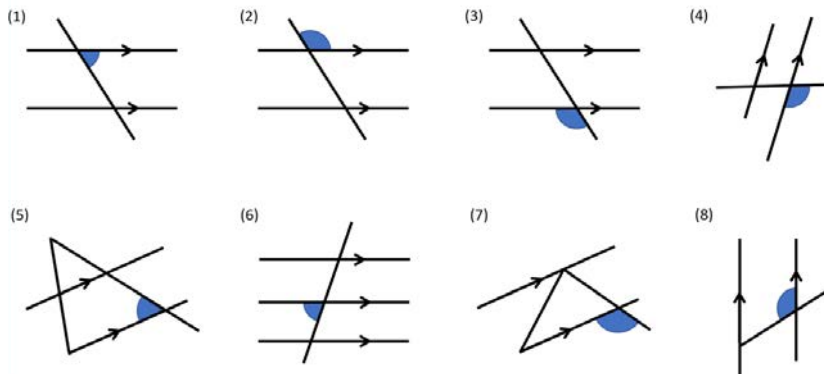
When a transversal intersects a pair of parallel lines, then

- Each pair of corresponding angles is equal.
- Each pair of vertically opposite angles is equal.
- Each pair of alternate interior angle is equal.
- Each pair of alternate exterior angle is equal.
- Interior angles on the same side of the transversal are supplementary.

Next, the teacher will draw the following figures on the board and ask the students to identify pairs of alternate interior angles in each figure.



Then the teacher will draw figures below wherein one angle has been shaded in each. The students will have to mark and shade their corresponding angles.



The teacher will give students 5 minutes to attempt these two questions. Then she will discuss them on the board.

Reference

[www.mathdrills.com](http://www.mathdrills.com)

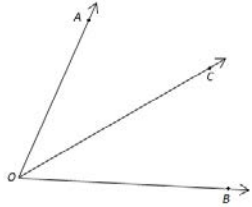
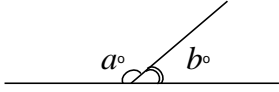
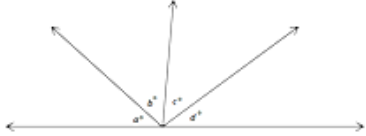
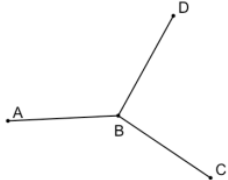
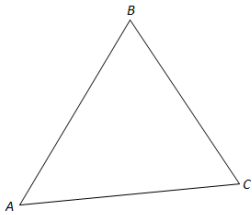
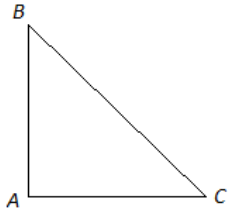
2

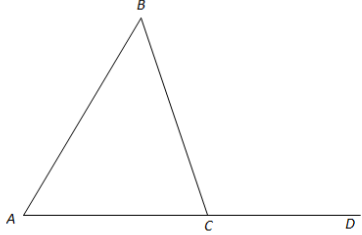

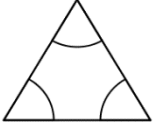
## Worksheet



The teacher will share the printed worksheet sheet on each desk. Students have to match the cells in column A with appropriate cells in column B.

S. No.	Diagram/Example	How to State as a Reason in an Exercise or a Proof
1	<p><math>a^\circ = b^\circ</math></p>	<p>“Consecutive adjacent angles on a line sum to <math>180^\circ</math>.”</p>

<p>2</p>	 $m\angle AOB = m\angle AOC + m\angle COB$	<p>“Acute angles in a right triangle sum to <math>90^\circ</math>.”</p>
<p>3</p>	 $a^\circ + b^\circ = 180$	<p>“Angle addition postulate”</p>
<p>4</p>	 $a^\circ + b^\circ + c^\circ + d^\circ = 180$	<p>“All angles in an equilateral triangle have equal measure.”</p>
<p>5</p>	 $m\angle ABC + m\angle CBD + m\angle DBA = 360^\circ$	<p>“Vertical angles are equal in measure.”</p>
<p>6</p>	 $m\angle A + m\angle B + m\angle C = 180^\circ$	<p>“The exterior angle of a triangle equals the sum of the two opposite interior angles.”</p>
<p>7</p>	 $m\angle A = 90^\circ; m\angle B + m\angle C = 90^\circ$	<p>“Base angles of an isosceles triangle are equal in measure.”</p>

8	 <p><math>m\angle BAC + m\angle ABC = m\angle BCD</math></p>	“Linear pairs form supplementary angles.”
9		“Angles at a point sum to 360°.”
10		“The sum of the angle measures in a triangle is 180°.”

# 7. TRIANGLES

**QR Code:**



**Learning outcome and Learning Objectives:**

Content area/Concepts	Learning Objectives	Learning Outcome
Congruence of Triangles	Observe the angles and sides of the given figures in order to show that they are congruent or not congruent	Applies axiomatic approach and derives proofs of mathematical statements particularly related to geometrical concepts, like parallel lines, triangles, quadrilaterals, circles etc. in order to solve problems using them
	Apply concepts of linear pairs of angles, vertically opposite angles, corresponding angles, alternate angles, transversal angles & exterior angles of a triangle in order to prove congruence between 2 triangles in a given figure	
Criteria for Congruence of Triangles	Illustrate the criteria of congruencies of triangles through diagrams (ASA, SAS, SSS, RHS) in order to prove relationships between given angles, sides and triangles of a given figure	
Some Properties of a Triangle	Apply criteria for congruence in a triangle with 2 congruent sides in order to prove that the angle opposite to the sides are equal and apply it in a given figure to solve for the measure of an angle	
Some More Criteria for Congruence of Triangles	Examine given triangles that satisfy AAA or SSA criteria in order to comment whether they are congruent	
Inequalities in a Triangle	Using properties of inequalities in triangles prove the relationship between any given sides or angles in a given figure	

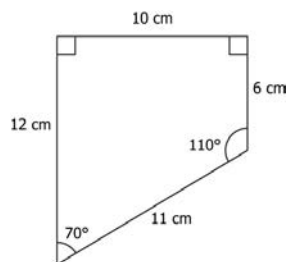


# Test items



**LOB:** Observe the angles and sides of the given figures in order to show that they are congruent or not congruent

1. A figure is shown.



Which of these figures is congruent to the given figure?

- a.
- b.
- c.
- d.

**Correct Answer:** Option a

2. Consider the following two statements about the rectangles a student draws.

Statement 1: Rectangles have the same perimeter.

Statement 2: Rectangles have the same area.

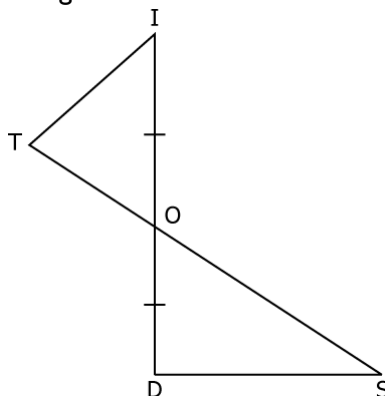
Which of the following statements is sufficient to conclude that the rectangles are congruent?

- Statement 1 alone is sufficient, but statement 2 alone is not sufficient.
- Statement 2 alone is sufficient, but statement 1 alone is not sufficient.
- Both statements together are sufficient, but neither statement alone is sufficient.
- Both statements together are not sufficient.

**Correct Answer:** Option c

**LOB:** Apply concepts of linear pairs of angles, vertically opposite angles, corresponding angles, alternate angles, transversal angles & exterior angles of a triangle in order to prove congruence between 2 triangles in a given figure

1. Consider the triangles made by two intersecting lines as shown.

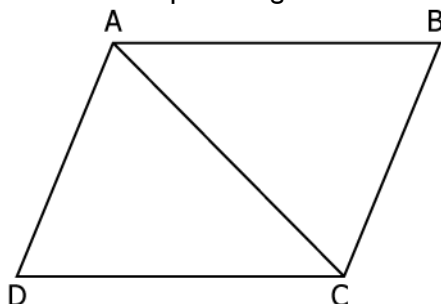


What additional information is required to prove that  $\Delta TOI \cong \Delta SOD$ ?

- $\angle DOS = \angle TOI$
- $\angle OTI = \angle ODI$
- $TO = OS$
- $TI = DS$

**Correct Answer:** Option c

2. Consider the parallelogram as shown.



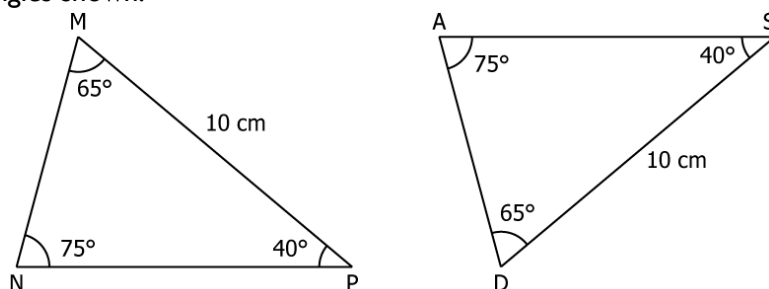
Based on the given information, can it be concluded that  $\Delta DAC \cong \Delta BCA$ ? Why or why not?

- Yes, because  $AB = CD$ ,  $AD = BC$ ,  $\angle DAC = \angle ACB$ ,  $\angle DCA = \angle CAB$ ,  $\angle ADC = \angle ABC$  and  $AC = AC$ .
- Yes, because  $AB = CD$ ,  $AD = BC$ ,  $\angle DAC = \angle CAB$ ,  $\angle DCA = \angle ACB$ ,  $\angle ADC = \angle ABC$  and  $AC = AC$ .
- No, because angle measures are not known.
- No, because side lengths are not known.

**Correct Answer:** Option a

**LOB:** Illustrate the criteria of congruencies of triangles through diagrams (ASA, SAS, SSS, RHS) in order to prove relationships between given angles, sides and triangles of a given figure

1. Consider the triangles shown.



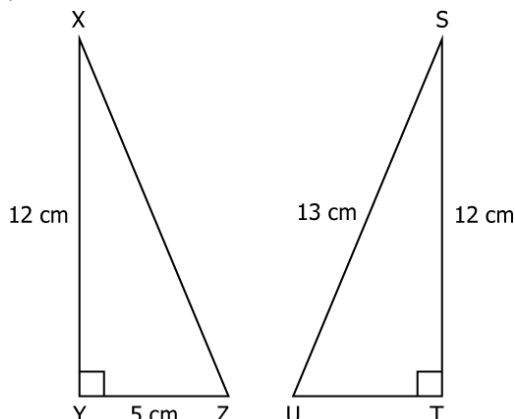
Which congruency criteria can be used to show that triangles are congruent?

- SSS

- b. SAS
- c. AAA
- d. ASA

**Correct Answer:** Option d

2. Consider the triangles shown.



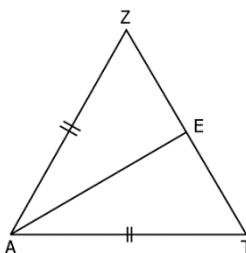
Which of these is not true about the given triangles?

- a.  $\triangle XYZ \cong \triangle STU$  (by SSS congruence rule).
- b.  $\triangle XYZ \cong \triangle STU$  (by RHS congruence rule).
- c.  $\triangle XYZ \cong \triangle STU$  (by ASA congruence rule).
- d.  $\triangle XYZ \cong \triangle STU$  (by SAS congruence rule).

**Correct Answer:** Option c

**LOB:** Apply criteria for congruence in a triangle with 2 congruent sides in order to prove that the angle opposite to the sides are equal and apply it in a given figure to solve for the measure of an angle

1. In triangle ZAT,  $AZ = AT$  and AE bisects  $\angle ZAT$ .

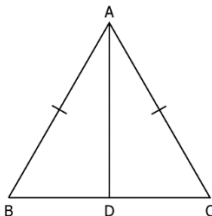


If the measure of  $\angle ZAT$  is  $50^\circ$ , which angle measures  $25^\circ$ ?

- a.  $\angle ZAE$
- b.  $\angle ZTA$
- c.  $\angle AET$
- d.  $\angle AEZ$

**Correct Answer:** Option a

2. In the triangle below, AD is the bisector of  $\angle A$  and  $AB = AC$ .



Which option has words that correctly complete the statement below?

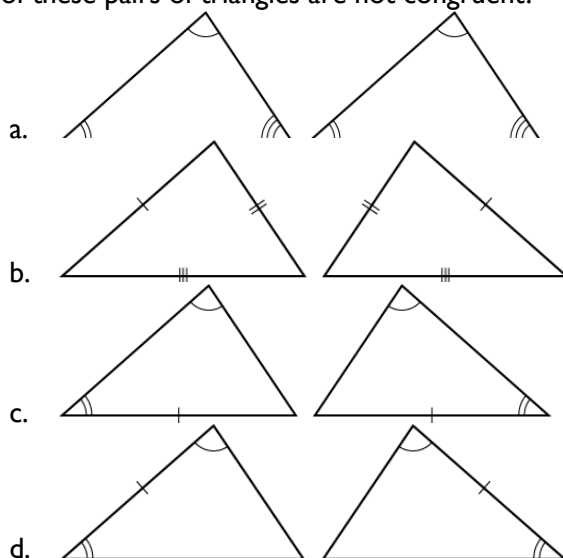
By \_\_\_\_\_ congruency criteria,  $\triangle ABD \cong \triangle ACD$  and using CPTCT, we get  $\angle ABC =$  \_\_\_\_\_.

- a. ASA;  $\angle ACD$
- b. SAS;  $\angle ACD$
- c. ASA;  $\angle ADC$
- d. SAS;  $\angle ADC$

**Correct Answer:** Option b

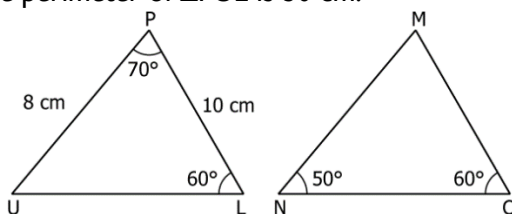
**LOB:** Examine given triangles that satisfy AAA or SSA criteria in order to comment whether they are congruent

1. Which of these pairs of triangles are not congruent?



**Correct Answer:** Option a

2. Two triangles are shown. The perimeter of  $\triangle PUL$  is 30 cm.



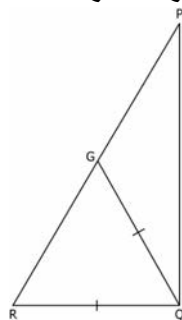
Are the triangles congruent?

- Yes, as on calculating the missing angle in each triangle it can be concluded that the triangles are congruent by AAA criteria.
- No, as the missing angle in each triangle cannot be calculated.
- Conclusion about the congruency of triangles can be made provided the length of the side NQ of triangle MNQ is known.
- Conclusion about the congruency of triangles can be made provided the length of the side MN or side MQ of triangle MNQ is known.

**Correct Answer:** Option d

**LOB:** Using properties of inequalities in triangles prove the relationship between any given sides or angles in a given figure

1. In the figure shown, G is a point on PR such that  $QG = QR$ .



Which option shows the correct steps to find the relationship between  $\angle QPR$  and  $\angle QRP$ ?

- Step 1:  $\angle QRG = \angle QGR$   
Step 2:  $\angle QPG + \angle PQG = \angle QGR$   
Step 3:  $\angle QPG + \angle PQG = \angle QRG$   
Step 4:  $\angle QPR < \angle QRP$
- Step 1:  $\angle RQG = \angle QGR$

Step 2:  $\angle QPG + \angle PQG = \angle QGR$

Step 3:  $\angle QPG + \angle PQG = \angle RQG \Rightarrow \angle QPG < \angle RQG$

Step 4:  $\angle QPR < \angle QRP$

c. Step 1: As  $QG = QR$ ,  $RQ < PQ$

Step 2:  $\angle QPG > \angle RQG$

Step 4:  $\angle QPR < \angle QRP$

d. Step 1: As  $QG = QR$ ,  $\angle QRG = \angle QGR$

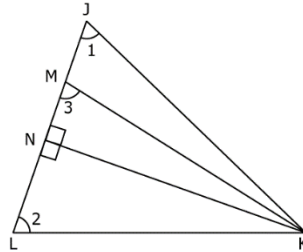
Step 2:  $\angle QPG + \angle PQG = \angle QGR$

Step 3:  $\angle QPG + \angle PQG = \angle QRG \Rightarrow \angle QPG < \angle QRG$

Step 4:  $\angle QPR > \angle QRP$

**Correct Answer:** Option a

2. In the given triangle,  $LN = MN$ .



If  $JK = 13$  cm, which could be the length of  $LK$ ?

- a. 9 cm
- b. 13 cm
- c. 17 cm
- d. 26 cm

**Correct Answer:** Option a

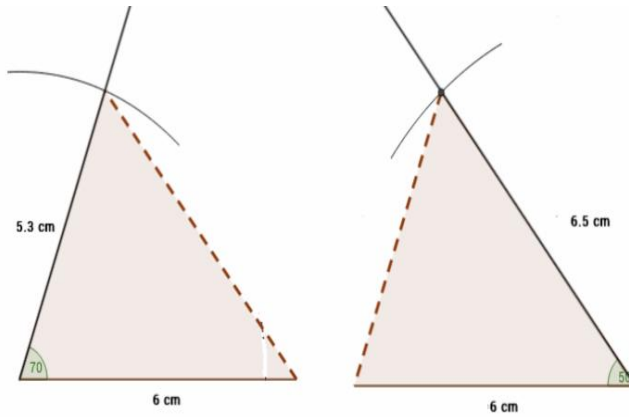
# Suggested Teacher Resources



## Lesson Plan



<b>Objective</b>	To illustrate the criteria of congruence of triangles through diagrams (ASA, SAS, SSS, RHS) in order to prove relationships between given angles, sides and triangles of a given figure
<b>Prerequisite</b>	Construction of triangles, ASA, SSS, SAS, RHS Congruence
<b>Vocabulary words</b>	Congruence
<b>Materials required</b>	Chart paper, markers, geometry box
<b>Procedure</b>	<p>The teacher will start the class by asking the following questions:            What do we mean when we say that two geometric figures are congruent? (congruent figures are of the same shape and size) What is the same in these two triangles? (All three corresponding sides and all three corresponding angles are equal?)</p> <p>Next, the teacher will draw the following figure on the board:</p> <div style="text-align: center;"> </div> <p>The teacher will ask the students to draw a triangle that is congruent to triangle ABC. Next, the teacher will give the following instructions:            You will all start by drawing a line segment that has the same length as the side AB. Then you will think about how many sides and angles you need to know in order to draw a triangle congruent to triangle ABC. You have ten minutes to work on this problem. Use your rulers, compasses and protractors.            Next, the teacher will discuss with the class how they decided where the third vertex of the triangle should be placed.            The teacher will discuss the following points:</p> <ul style="list-style-type: none"> <li>• If we know that three sides in one triangle are the same lengths as three sides in another triangle then the two triangles must be congruent. (SSS congruence criterion)</li> <li>• If two angles and the included side in one triangle are the same as two angles and the included side in another triangle, then the two triangles must be congruent. (ASA congruence criterion)</li> <li>• If two sides and the angles between them (included angle) in one triangle are equal to two sides and the included angle in another triangle then the two triangles must be congruent. (SAS congruence criterion)</li> </ul>



- Let's think about what will happen if we take the 5.3 cm and 6 cm sides and the non-included angle of 50 degrees.
- Could we have the SSA axiom? Given two sides and a non-included angle, how many different triangles can you make?  
At least two such triangles are possible.
- Could we get away without any side length? Let's take the three angles, 50°, 60° and 70°. How many different triangles can you make using these angles? (Everyone could have a different triangle.) Could we have the AAA axiom?  
No.

The teacher will ask students to sum up their conclusions from this activity about the SSS, SAS and ASA congruence criterion.

- The teacher could use a similar approach for the RHS congruence criterion

Source

<https://www.projectmaths.ie/documents/MathsCounts2016/CongruentTriangles.pdf>

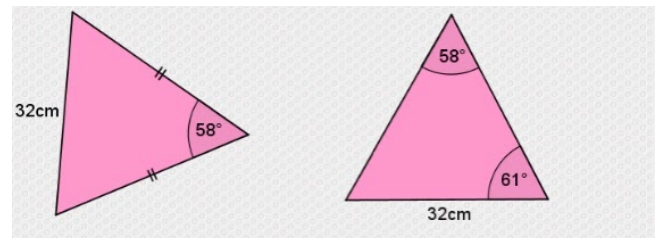
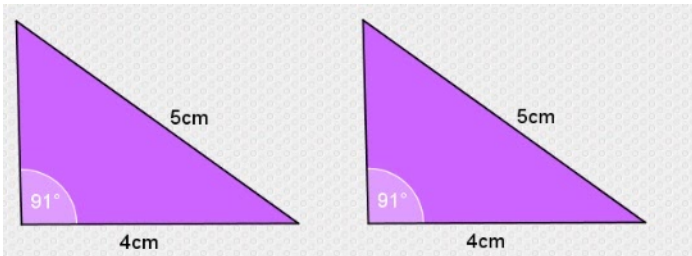
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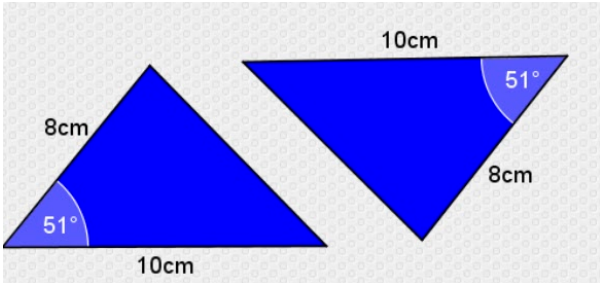
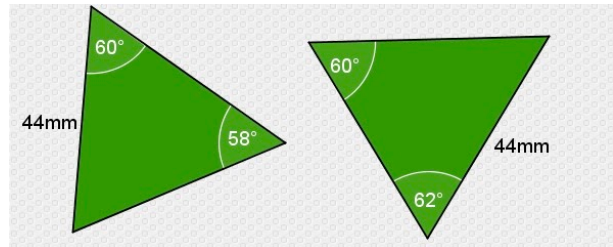
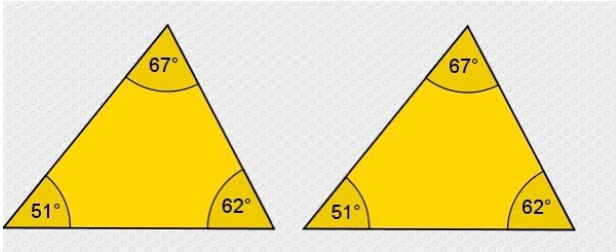
## Activity



The teacher will divide the class into groups and distribute sheets with the following pairs of triangles printed on them.

The teacher would then ask the students to identify the type of congruence which can be used to prove the two triangles congruent. Students have to attempt this along with their partners and give reasons. The teacher would ask volunteers to come at the board and discuss for each pair.





Source: transum.org



# 8. QUADRILATERALS

**QR Code:**



**Learning outcome and Learning Objectives:**

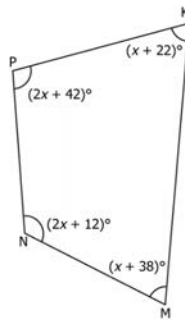
Content area/Concepts	Learning Objectives	Learning Outcome
Angle sum property of a Quadrilateral	Apply angle sum property of quadrilateral in order to find the value of the unknown angle	Applies axiomatic approach and derives proofs of mathematical statements particularly related to geometrical concepts, like parallel lines, triangles, quadrilaterals, circles etc. in order to solve problems using them
Types of Quadrilaterals	List the properties of quadrilaterals in order to classify real life objects into different types of Quadrilaterals	
Properties of Parallelogram	List the properties of parallelogram in order to identify if a given quadrilateral is a parallelogram	
Another Condition for a Quadrilateral to be a Parallelogram	Apply properties of parallelogram in order to find a) an unknown angle b) an unknown side	
The Mid-point Theorem	Prove the midpoint theorem of triangles using concepts of congruency and transversal angles in order to extend the application to quadrilaterals	

# Test items



**LOB:** Apply angle sum property of quadrilateral in order to find the value of the unknown angle

1. A quadrilateral PKMN is shown below.



What is the measure of  $\angle NPK$ ?

- $124^\circ$
- $104^\circ$
- $84^\circ$
- $64^\circ$

**Correct Answer:** Option a

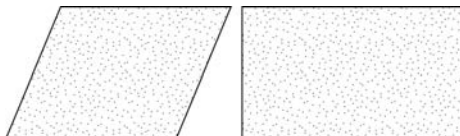
2. In quadrilateral BDGH, if  $\angle BDG = 2\angle DGH$  and  $\angle BHG = 3\angle HBD$ , which of the following is true about  $\angle BDG$ ?

- $\angle DGH = \frac{1}{3} (360^\circ - 3\angle HBD)$
- $\angle DGH = \frac{1}{2} (360^\circ - 4\angle HBD)$
- $\angle DGH = \frac{1}{3} (360^\circ - 4\angle HBD)$
- $\angle DGH = \frac{1}{2} (360^\circ - 3\angle HBD)$

**Correct Answer:** Option a

**LOB:** List the properties of quadrilaterals in order to classify real life objects into different types of Quadrilaterals

1. Ravi cut two pieces of marble as shown.

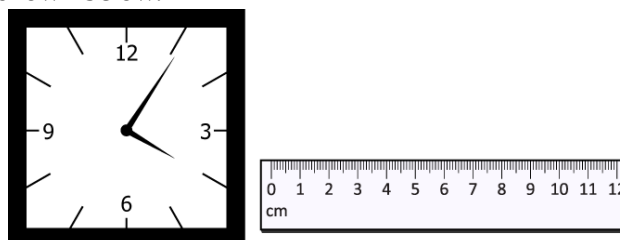


What is common about the shapes of both the pieces?

- Both are squares.
- Both are rhombus.
- Both are rectangles.
- Both are parallelograms.

**Correct Answer:** Option d

2. A clock and a scale are shown below.



Arjun claims that the clock shown is a square but not a rhombus and Vinod claims that the ruler shown is a rectangle but not a parallelogram.

Whose claim is/are correct?

- a. Only Arjun
- b. Only Vinod
- c. Both of them
- d. None of them

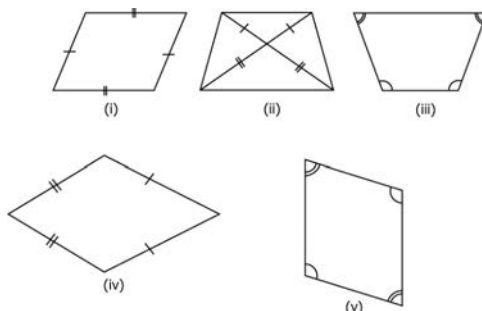
**Correct Answer:** Option d

**LOB:** List the properties of parallelogram in order to identify if a given quadrilateral is a parallelogram

1. Which of the following is NOT a property of a quadrilateral that is a parallelogram?
  - a. Diagonals of a quadrilateral bisect each other
  - b. A pair of adjacent sides of a quadrilateral is equal
  - c. Each pair of opposite sides of a quadrilateral is equal
  - d. Each pair of opposite angles of a quadrilateral is equal

**Correct Answer:** Option b

2. Some quadrilaterals are shown below.



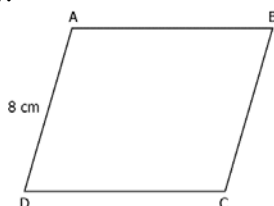
Which of the following quadrilaterals are parallelograms?

- a. Only i and v
- b. Only i, ii and v
- c. Only ii, iii and iv
- d. Only ii, iv and v

**Correct Answer:** Option a

**LOB:** Apply properties of parallelogram in order to find a) an unknown angle b) an unknown side

1. A parallelogram ABCD is shown below.

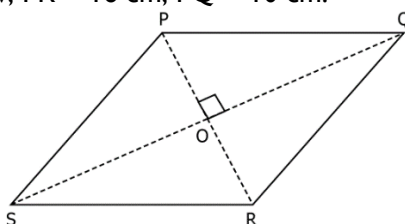


If the perimeter of the parallelogram is 36 cm, what is the length of AB?

- a. 5 cm
- b. 8 cm
- c. 10 cm
- d. 12 cm

**Correct Answer:** Option c

2. In the parallelogram shown below,  $PR = 16$  cm,  $PQ = 10$  cm.



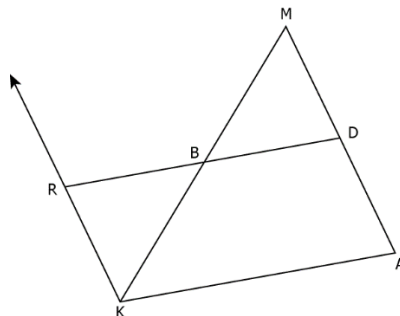
What is the length of the diagonal SQ?

- a. 6 cm
- b. 8 cm
- c. 12 cm
- d. 16 cm

**Correct Answer:** Option c

**LOB:** Prove the midpoint theorem of triangles using concepts of congruency and transversal angles in order to extend the application to quadrilaterals

1. A figure is shown below where B and D are midpoints of sides MK and MA. Danny constructs a ray KR such that  $MA \parallel KR$  to prove the midpoint theorem.

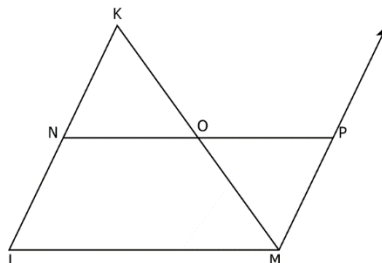


He proves  $\triangle MBD$  is congruent to  $\triangle KBR$  by ASA congruency. Which of the following is the next step in the proof of the midpoint theorem?

- a. show that  $BD = RB$
- b. show that  $BD = BK$
- c. show that  $MB = RK$
- d. show that  $MD = BK$

**Correct Answer:** Option a

2. In the figure shown, Points N and O are midpoints of sides KL and KM of  $\triangle KLM$ . Ananya wants to prove  $NO \parallel LM$ . She constructs a ray MP such that  $KL \parallel MP$ .



She first proves  $\triangle KON \cong \triangle MOP$ . Which of the following justifies her step of proof?

- a.  $\triangle KON \cong \triangle MOP$  by SAS congruency because  $KO = OM$ ,  $NO = OP$  and  $\angle KON = \angle MOP$ .
- b.  $\triangle KON \cong \triangle MOP$  by SAS congruency because  $KO = OM$ ,  $KN = MP$  and  $\angle NKO = \angle PMO$ .
- c.  $\triangle KON \cong \triangle MOP$  by ASA congruency because  $NO = OP$ ,  $\angle KON = \angle MOP$  and  $\angle NKO = \angle PMO$ .
- d.  $\triangle KON \cong \triangle MOP$  by ASA congruency because  $KO = OM$ ,  $\angle KON = \angle MOP$  and  $\angle NKO = \angle PMO$ .


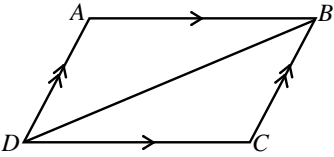
**Correct Answer:** Option d

# Suggested Teacher Resources



## Lesson Plan



Objective	Use triangle congruence criteria in order to demonstrate why certain properties of parallelograms hold true
Prerequisite	Properties of parallelograms, triangle congruence criteria.
Materials required	White sheet, tracing sheet, geometry box
Procedure	<p>The teacher will begin by asking the students to define a parallelogram.</p> <p>A parallelogram is a quadrilateral in which both pairs of opposite sides are parallel. Some properties of parallelogram that we have studied in previous classes are:</p> <ol style="list-style-type: none"> <li>1. Opposite sides are equal.</li> <li>2. Opposite angles are equal.</li> <li>3. Diagonals bisect each other.</li> </ol>  <p>The teacher will bring the attention of students by announcing that now they will be examining why each of these properties is true.</p> <p>Let us begin by proving the properties 1 and 2 together.</p> <p>If a quadrilateral is a parallelogram, then its opposite sides and angles are equal in measure.</p> <p>The teacher will write on the blackboard that let us say we are given a parallelogram ABCD. We will now prove that its opposite sides and angles are equal.</p> <p>The teacher will draw a figure and note the Given and to prove as below:</p> <p>Given: ABCD is a parallelogram (<math>\overline{AB} \parallel \overline{CD}</math>, <math>\overline{AD} \parallel \overline{CB}</math>)</p> <p>To prove: <math>AD = CB</math>, <math>AB = CD</math>, <math>m\angle A = m\angle C</math>, <math>m\angle B = m\angle D</math></p>  <p>A construction is required here to draw diagonal <math>\overline{BD}</math>.</p> <p>We will now begin the proof.</p> <p>In parallelogram ABCD,</p> <p><math>AB \parallel DC</math> and <math>BD</math> is a transversal.</p> <p>We know that when parallel lines are cut by a transversal, then alternate interior angles are equal in measure.</p> $m\angle ABD = m\angle CDB$

$$m\angle CBD = m\angle ADB$$

$$BD = DB \text{ (common)}$$

Therefore, by ASA congruence criterion, we can say that,

$$\triangle ABD \cong \triangle CDB$$

We know that the corresponding sides and angles of congruent triangles are equal in measure. Therefore,

$$AD = CB \dots (1)$$

$$AB = CD \dots (2)$$

$$m\angle A = m\angle C \dots (3)$$

Similarly, if we construct the diagonal AC, we can prove that

$$m\angle B = m\angle D \dots (4)$$

Therefore, by equations (1), (2), (3) and (4), we have proved that in parallelogram ABCD, pairs opposite sides and opposite angles are equal.

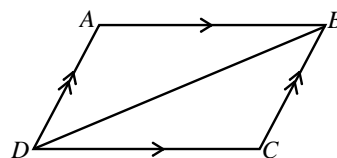
Let us now consider the property 3.

If a quadrilateral is a parallelogram, then the diagonals bisect each other.

The teacher will mention the Given and to prove as below:

Given: ABCD is a parallelogram,  $AD = CB$ ,  $AB = CD$ ,  $m\angle A = m\angle C$ ,  $m\angle B = m\angle D$

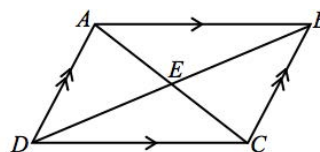
To prove: diagonals bisect each other,  $AE = CE$ ,  $DE = BE$



We need to do a construction here. Draw the diagonal AC.

In parallelogram ABCD,

Since  $AB \parallel DC$  and AC is a transversal,



We know that if parallel lines are cut by a transversal, then alternate interior angles are equal in measure.

Therefore,  $m\angle BAC = m\angle DCA$

$m\angle AEB = m\angle CED$  (vertically opposite angles are equal.)

$AB = CD$  (Opposite sides of a parallelogram are equal in length)

Therefore, by AAS congruence criterion, we can say,

$$\triangle AEB \cong \triangle CED$$

We know that corresponding sides of congruent triangles are equal in length.

Hence,  $AE = CE$ ,  $DE = BE$

We have now proved that the diagonals of a parallelogram bisect each other.

This way by using triangle congruence criterion, we have proved the properties of parallelogram. There are special cases of parallelogram like rectangle, rhombus and square. The properties of parallelogram hold true for them as well.

We know that the diagonals of a rectangle are equal in length. The teacher could then ask students to give a proof of the same by following a similar approach as above.

Source

<https://www.engageny.org/resource/geometry-module-1-topic-e-lesson-28>

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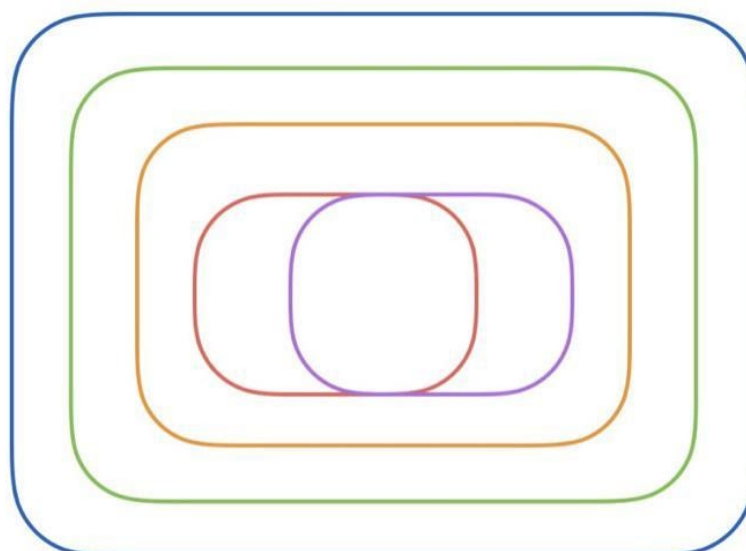
### Activity



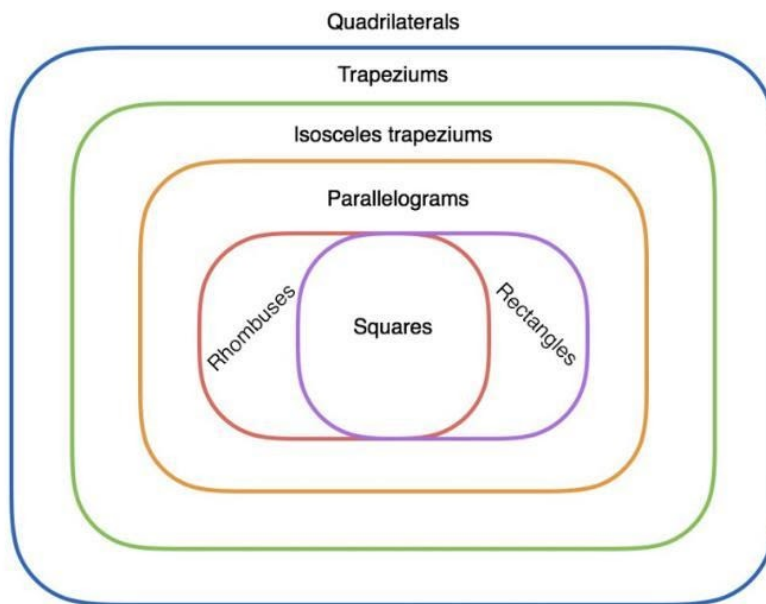
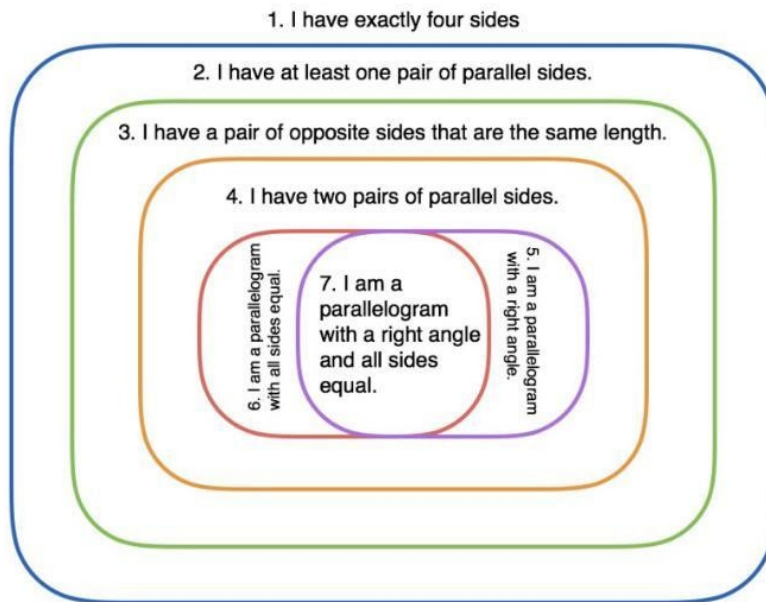
The teacher will write the following categories on the board and would ask students to make a Venn Diagram for the following categories and also name them.

<p><b>1</b> I have exactly four sides.</p>	<p><b>5</b> I am a parallelogram with a right angle.</p>
<p><b>2</b> I have at least one pair of parallel sides.</p>	<p><b>6</b> I am a parallelogram with all sides equal.</p>
<p><b>3</b> I have a pair of opposite sides that are the same length.</p>	<p><b>7</b> I am a parallelogram with a right angle and all sides equal.</p>
<p><b>4</b> I have two pairs of parallel sides.</p>	

The teacher will ask the students to fill these categories in the following diagram:



Solution:



Source: projectmaths.ie



# 9. AREA OF PARALLELOGRAM & TRIANGLES

**QR Code:**



**Learning outcome and Learning Objectives:**

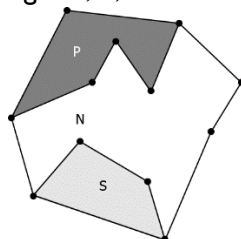
Content area/Concepts	Learning Objectives	Learning Outcome
Figures on the same Base and Between the same Parallels	Identify the planar region and area associated in order to show that area of non-overlapping planar region formed is the sum of their areas	Applies appropriate formulae in order to find areas of all types of triangles
	Identify if given figures lie on the same base and between the same parallels in order to write the common base and the two parallels	
Parallelograms on the same Base and between the same Parallels	Extend the understanding of congruency of triangle in order to prove that: Parallelograms on the same base and between the same parallels are equal in area	
Triangle & Parallelogram on the same Base and between the same parallels	Extend prior knowledge from this chapter in order to prove that when a triangle and a parallelogram are on the same base and between the same parallels, the area of the triangle is equal to half the area of the parallelogram	
Triangles on the same Base and between the same parallels	Extend prior knowledge in order to prove that Two triangles on the same base (or equal bases) and between the same parallels are equal in area	
	Extend prior knowledge to prove that Two triangles having the same base (or equal bases) and equal areas lie between the same parallels.	

# Test items



**LOB:** Identify the planar region and area associated in order to show that area of non-overlapping planar region formed is the sum of their areas

1. The figure shown is composed of three regions, P, N and S as shown.

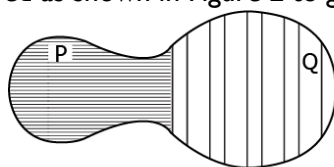


Which of these gives the area of the figure?

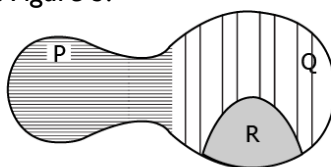
- The area of the largest region.
- The sum of the areas of the three regions.
- The sum of the areas of the largest and the smallest region.
- The difference between the area of the largest and the smallest region.

**Correct Answer:** Option b

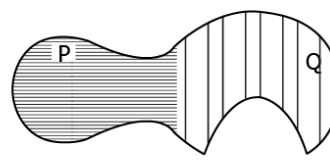
2. Figure 1 shown below is composed of two regions, P and Q. Part of the region Q (shown as region R) is removed as shown in Figure 2 to get Figure 3.



**Figure 1**



**Figure 2**



**Figure 3**

Area of Region P =  $p$

Area of Region Q =  $q$

Area of Region R =  $r$

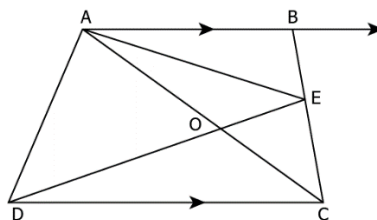
Which expression gives the area of Figure 3?

- $p + q + r$
- $p + q + 2r$
- $p + q - r$
- $p + q - 2r$

**Correct Answer:** Option c

**LOB:** Identify if given figures lie on the same base and between the same parallels in order to write the common base and the two parallels

1. Consider the figure shown.

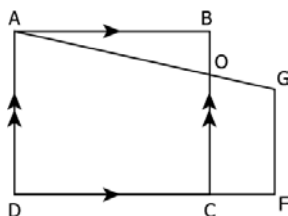


Which set of shapes lie on the same base CD and between the same parallels?

- ABCD and  $\triangle ADE$
- ABCD and  $\triangle ADC$
- AECD and  $\triangle ADE$
- AECD and  $\triangle ADC$

**Correct Answer:** Option b

2. Consider the figure as shown.



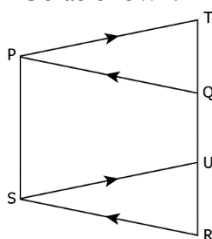
If segment BD is constructed, how many pair(s) of figures will lie on the base CD and between the same parallels AB and CD?

- a. 3
- b. 4
- c. 5
- d. 1

**Correct Answer:** Option d

**LOB:** Extend the understanding of congruency of triangle in order to prove that Parallelograms on the same base and between the same parallels are equal in area

1. Consider two parallelograms PQRS and PTUS as shown.

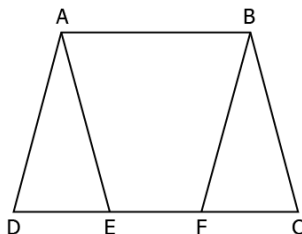


Which of these helps proving that the parallelograms have the same area?

- a.  $\triangle PTQ \cong \triangle SUR$  using ASA congruency rule
- b.  $\triangle PTQ \cong \triangle SRU$  using SAS congruency rule
- c.  $\text{ar}(\text{SUR}) = \text{ar}(\text{PQUS})$
- d.  $\text{ar}(\text{PQUS}) = 2 \text{ar}(\text{PQT})$

**Correct Answer:** Option a

2. In the figure below, parallelogram ABFD and ABCE are constructed such that  $\triangle BFC$  is congruent to  $\triangle ADE$  as shown.



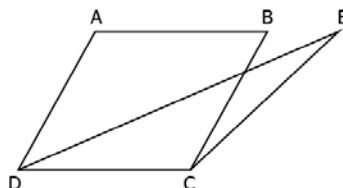
Which of these shows the relation between the areas of the parallelograms?

- a.  $\text{ar}(\text{ABFD}) = \text{ar}(\text{ABCE})$  as  $\text{ar}(\triangle BFC) = \text{ar}(\triangle ADE)$  and area AEBF is common
- b.  $\text{ar}(\text{ABFD}) < \text{ar}(\text{ABCE})$  as  $\text{ar}(\triangle BFC) > \text{ar}(\triangle ADE)$  and area AEBF is common
- c.  $\text{ar}(\text{ABFD}) > \text{ar}(\text{ABCE})$  as  $\text{ar}(\triangle BFC) < \text{ar}(\triangle ADE)$  and area AEBF is common
- d.  $\text{ar}(\text{ABFD}) = \text{ar}(\text{ABCE}) - \text{ar}(\text{AEBF})$  as  $\text{ar}(\triangle BFC) = \text{ar}(\triangle ADE)$

**Correct Answer:** Option a

**LOB:** Extend prior knowledge from this chapter in order to prove that when a triangle and a parallelogram are on the same base and between the same parallels, the area of the triangle is equal to half the area of the parallelogram

1. The parallelogram ABCD and  $\triangle DEC$  are on the same base and between the same parallels as shown.

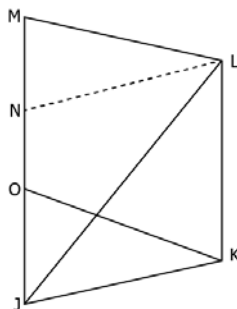


Which relationship is correct?

- a. Option a:  $\text{ar}(\text{ABCD}) = \frac{1}{2} \text{ar}(\Delta \text{DEC})$
- b. Option b:  $\text{ar}(\text{ABCD}) = 2\text{ar}(\Delta \text{DEC})$
- c. Option c:  $\text{ar}(\text{ABCD}) = \frac{1}{3} \text{ar}(\Delta \text{DEC})$
- d. Option d:  $\text{ar}(\text{ABCD}) = \text{ar}(\Delta \text{DEC})$

**Correct Answer:** Option b

2. In the figure shown, KLMO is a parallelogram.



2.

Tina's work to find the relation between the areas of  $\Delta \text{JKL}$  and parallelogram KLMO is given.

**Step 1:**  $\text{ar}(\Delta \text{JKL}) = 2 \text{ar}(\Delta \text{JLN})$  as JKN is a parallelogram

**Step 2:**  $\text{ar}(\Delta \text{JKL}) = \frac{1}{3} \text{ar}(\text{JKLN})$

**Step 3:**  $\text{ar}(\text{JKLN}) = \text{ar}(\text{KLMO}) \Rightarrow \text{ar}(\Delta \text{JKL}) = \frac{1}{3} \text{ar}(\text{KLMO})$

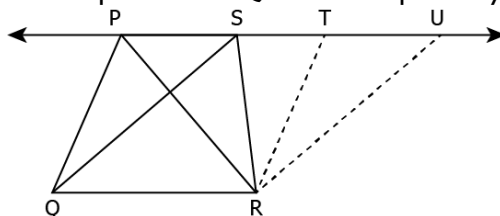
In which step did Tina make the first error and what is the correct step?

- a. Step 2;  $3 \text{ar}(\Delta \text{JKL}) = \text{ar}(\text{JKLN})$
- b. Step 3;  $\text{ar}(\text{JKLN}) = 2 \text{ar}(\text{KLMO})$
- c. Step 1;  $\text{ar}(\Delta \text{JKL}) = \text{ar}(\Delta \text{JLN})$
- d. Step 3;  $\text{ar}(\text{JKLN}) = \frac{1}{2} \text{ar}(\text{KLMO})$

**Correct Answer:** Option c

**LOB:** Extend prior knowledge in order to prove that Two triangles on the same base (or equal bases) and between the same parallels are equal in area

1. Line segments TR and UR are drawn parallel to PQ and SR respectively.

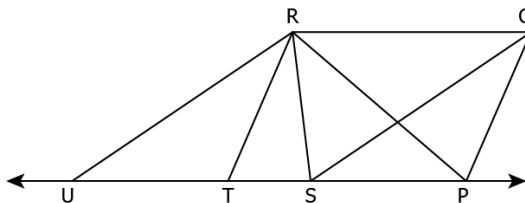


Which of these is the first step to prove that  $\text{ar}(\Delta \text{PQR}) = \text{ar}(\Delta \text{SQR})$ ?

- a.  $\text{ar}(\text{PQRT}) = \text{ar}(\text{SQRU})$  as they are on the same base and between the same parallels
- b.  $\text{ar}(\text{PQRT}) = 2 \text{ar}(\text{SQRU})$  as they are on the same base and between the same parallels
- c.  $\text{ar}(\text{PQRT}) = \text{ar}(\text{SQRU})$  as they are between the same parallels
- d.  $\text{ar}(\text{PQRT}) = 2 \text{ar}(\text{SQRU})$  as they are between the same parallels

**Correct Answer:** Option a

2. In the figure below, QR is parallel to PU.



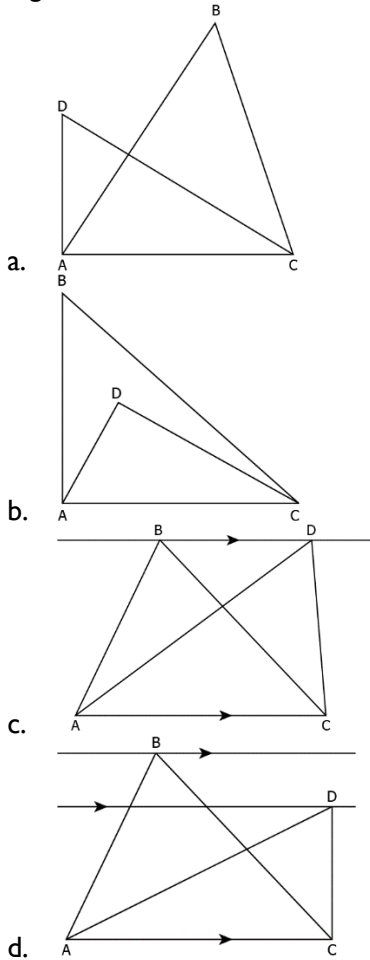
Which two triangles have equal areas?

- a.  $\Delta \text{PQR}$  and  $\Delta \text{SQR}$
- b.  $\Delta \text{RTP}$  and  $\Delta \text{RUS}$
- c.  $\Delta \text{RPS}$  and  $\Delta \text{RUS}$
- d.  $\Delta \text{PQR}$  and  $\Delta \text{RUS}$

**Correct Answer:** Option a

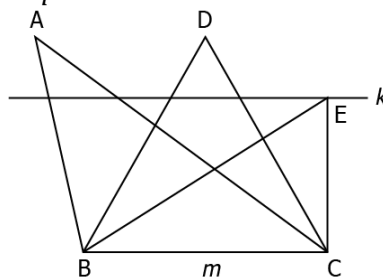
**LOB:** Extend prior knowledge to prove that Two triangles having the same base (or equal bases) and equal areas lie between the same parallels.

1. Two triangles  $\triangle ABC$  and  $\triangle ADC$  have the same base and equal areas. Which figure shows the triangles?



**Correct Answer:** Option c

2. In the figure, the area of  $\triangle BDC$  is  $2mp$ .



What additional information is required to prove that line passing through  $A$  and  $D$  is parallel to  $BC$ ?

- The height of the perpendicular from  $A$  on  $BC$  produced is  $2p$ .
- The length of side  $AB$  is  $4p$ .
- The height of the perpendicular from  $A$  on  $BC$  produced is  $4p$ .
- The length of side  $AB$  is  $2p$ .

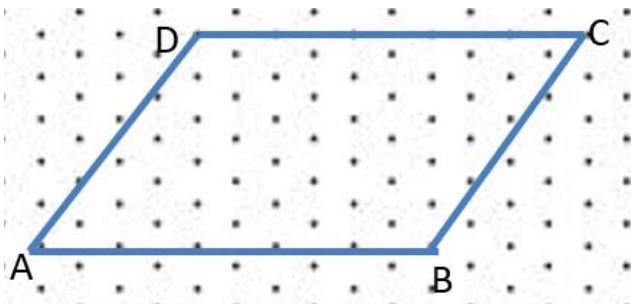
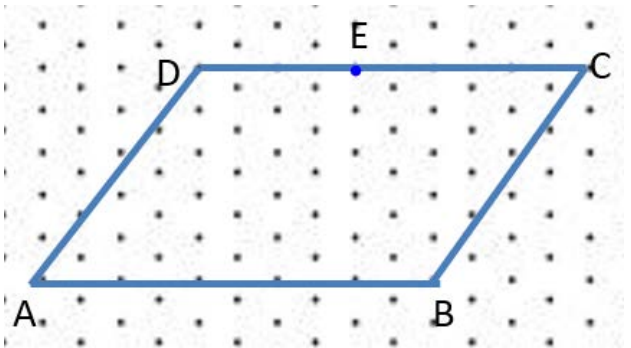
**Correct Answer:** Option c

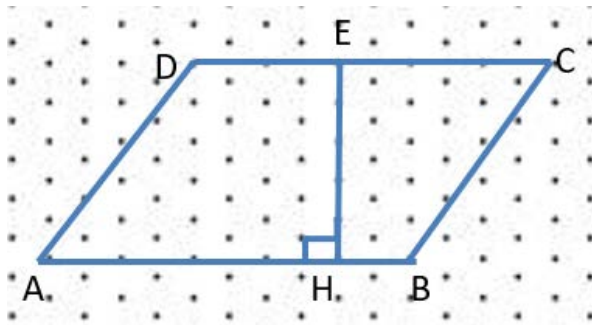
# Suggested Teacher Resources



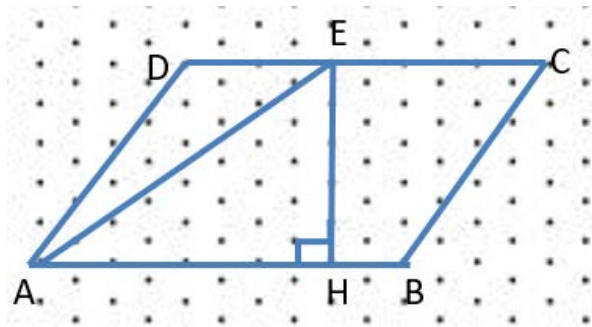
## Lesson Plan



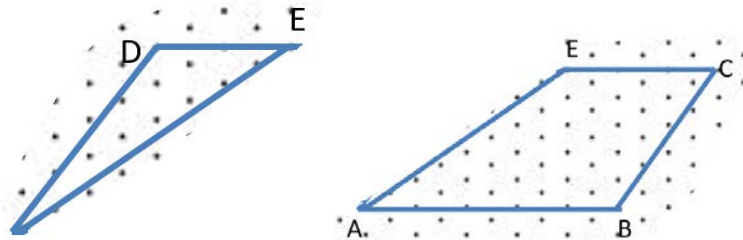
Objective	Extend the understanding of congruence of triangle in order to prove that: Parallelograms on the same base and between the same parallels are equal in area
Prerequisite	<ol style="list-style-type: none"> <li>1. Area of parallelogram = base <math>\times</math> height.</li> <li>2. Shortest distance between two parallel lines is the perpendicular distance between parallel lines and it remains the same for that pair of parallel lines.</li> </ol>
Vocabulary words	Congruence, parallelograms
Materials required	Isometric sheets, a pair of scissors, tape, ruler
Procedure	<p>The teacher will start the class by an activity. The teacher needs to ensure that each group has access to a pair of scissors, geometry box and isometric sheet. The students are supposed to do this activity with their partners or in groups 4 if resources are less. The teacher will guide the students with the steps. These are as follows:</p> <ol style="list-style-type: none"> <li>1. On the isometric sheet, draw a parallelogram. Name it as ABCD.</li> </ol>  <ol style="list-style-type: none"> <li>2. Mark any point E on DC.</li> </ol>  <ol style="list-style-type: none"> <li>3. Draw a perpendicular from point E to the opposite side AB.</li> </ol>



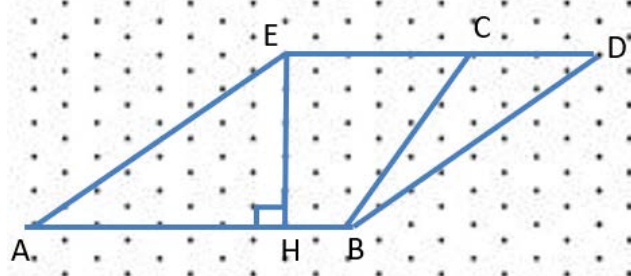
4. From vertex A draw a straight line till the point E (on DC) by using a ruler and pencil



5. Cut along AE to get  $\triangle AED$  and quadrilateral ABCE.



6. Paste the  $\triangle AED$  on the other side of the quadrilateral ABCE along the side BC.



7. We get a new parallelogram ABDE.

The teacher will now, divert student's attention to the following points:

- i. We observe that the two parallelograms ABCD and ABDE have the same base AB.
- ii. The two parallelograms lie between the same two parallel lines, i.e., AB and CD and height between them is same at all points.

By formula, area of a parallelogram = base x height

$$\text{ar}(\text{||gm ABCD}) = AB \times EH$$

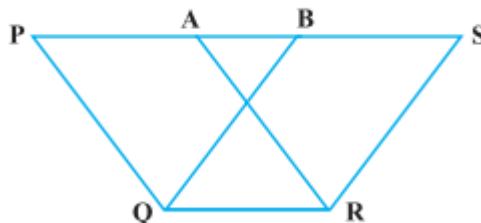
$$\text{ar}(\text{||gm ABDE}) = AB \times EH$$

This way the students will verify that two parallelograms lying on the same base and between the same parallel lines are equal in area.

The teacher will then mention that through this activity the theorem which states- 'Parallelograms on the same base and between the same parallel lines are equal in area' can be verified.

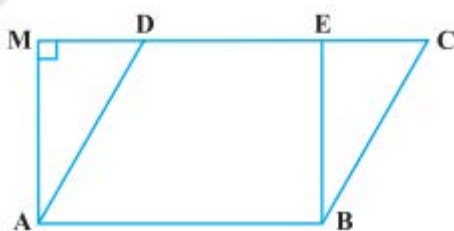
We know that the rectangle, rhombus and square are special cases of parallelograms. Hence, this theorem applies to them as well.

The teacher can then draw a few figures on the board and ask students to tell about the area of the figures.



In the above figure, Parallelograms QRAP and QRSB have the same base QR and are between the same parallel lines PS and QR.

Therefore,  $\text{ar}(\text{||gm QRAP}) = \text{ar}(\text{||gm QRSB})$ .



In the above figure, Rectangle ABEM and ABCD have the same base AB and are between the same parallel lines AB and MC.

Therefore,  $\text{ar}(\text{rectangle ABEM}) = \text{ar}(\text{||gm ABCD})$ .

The teacher can then ask a question, If the two rhombi are lying on the same base and between the same parallel lines and area of the one rhombus is  $364 \text{ cm}^2$ . What will be the area of second rhombus?

Here also, the theorem that parallelogram on the same base and between the same parallels are equal in area will be used.

If the area of one rhombus is  $364 \text{ cm}^2$

Then the area of the other rhombus will also be  $364 \text{ cm}^2$

The teacher can give a few more questions like above based on the same concept to the students to practice.

Source

<https://www.cbsepapers.com/cbse/cbse-class-9-maths-lab-manual-area-of-parallelograms-on-the-same-base>

2

## Activity



**Aim:** To verify that if a triangle and a parallelogram are on the same base and between the same parallels, then the area of the triangle is equal to half the area of the parallelogram.

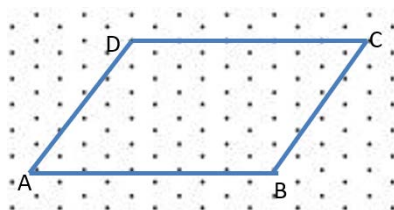
**Materials required:** Isometric sheets, ruler, pencil.



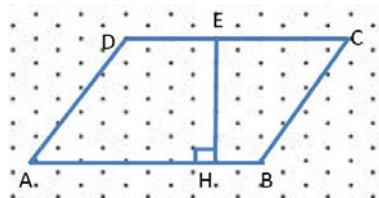
Procedure:

This will be similar to the activity done in the above lesson plan.

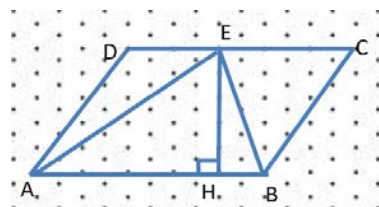
1. On an isometric sheet, draw a parallelogram ABCD.



2. Take a point E on the side DC, and draw an altitude EH.



3. Join AE and EB. We will get a triangle  $\triangle EAB$ .



4. We can observe that,
  - Parallelogram ABCD and the  $\triangle EAB$  both have the same base AB
  - Parallelogram ABCD and  $\triangle EAB$  lie between the same parallels AB and DC, i.e., they have the same altitude (or we can say height).
5. Let us assume, that the distance between each dot on the isometric sheet is 1 cm. Then let us try to find the area of the parallelogram ABCD and the  $\triangle EAB$ .
6. We can see that,  
 In Parallelogram ABCD,  
 Base= AB= 5cm  
 Height=EH=5 cm  
 $\text{ar(Parallelogram ABCD)} = \text{Base} \times \text{Height} = 5\text{cm} \times 5\text{cm} = 25\text{cm}^2$
7. In triangle  $\triangle EAB$ ,  
 Base= AB= 5cm  
 Height=EH=5 cm  
 $\text{ar}(\triangle EAB) = \frac{1}{2} \text{Base} \times \text{Height} = \frac{1}{2} 5\text{cm} \times 5\text{cm} = 12.5\text{cm}^2$
8. We can see that, here the area of  $\triangle EAB = 1/2 \text{ ar(Parallelogram ABCD)}$ .
9. The same activity can be repeated again where the students draw a parallelogram and triangle on the same base and between the same parallels but of different measurements.
10. This way it could be verify that if a parallelogram and a triangle are on the same base and between the same parallel lines, then the area of the triangle will be equal to half the area of the parallelogram.

# 10. CIRCLES

**QR Code:**



**Learning outcome and Learning Objectives:**

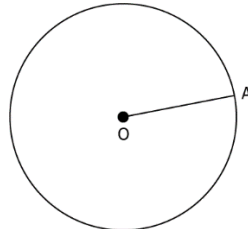
Content area/Concepts	Learning Objectives	Learning Outcome
Circles and its Related Terms: A Review	Construct a circle of a given radius in order to verify that the length of multiple segments drawn from the centre of the circle to the circumference is equal	Applies axiomatic approach and derives proofs of mathematical statements particularly related to geometrical concepts, like parallel lines, triangles, quadrilaterals, circles etc. in order to solve problems using them
	Define radius, chord, diameter, segment (major and minor), arc (major and minor), interior or exterior of a circle in order to illustrate and label them on a given circle	
Angle Subtended by a Chord at a Point	Apply theorems regarding angle subtended by a chord in a circle in order to find the measure of an angle in the given figure	
Perpendicular from the Centre to a Chord	Apply the property of perpendicular from the centre to the chord in order to solve for the missing values (lengths and angles) in a given figure	
Circle through Three Points	Construct circle passing through 1, 2 & 3 non-collinear points in order to comment on how many circles can be constructed passing through them	
Equal Chords and their Distances from the Centre	Use the value of radius and perpendicular to the chord in order to compute the length of a chord	
Angle subtended by arc of the circle	Interpret and apply theorems on the angles subtended by arcs of a circle in order to solve for unknown values in given examples	
Cyclic Quadrilaterals	Apply the relation between angles of a cyclic quadrilateral in order to solve for the value of a given angle	

# Test items



**LOB:** Construct a circle of a given radius in order to verify that the length of multiple segments drawn from the centre of the circle to the circumference is equal

1. Given below is a circle with radius OA.

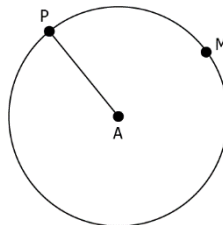


If P is any point on the circle, which of these is the correct relation between OA and OP?

- $OA = OP$
- $OA = 2 OP$
- $OA = \frac{1}{2}OP$
- $OA = \frac{1}{3}OP$

**Correct Answer:** Option a

2. Given below is a circular park with centre A. Madhav walks at a uniform speed of 0.5 m/s from Gate P and reaches the centre of the park in 150 seconds.



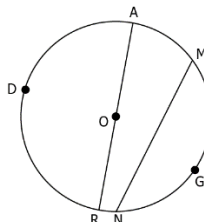
What is the straight-line distance between the centre of the park and Gate M?

- 300 m
- 150 m
- 2.5 m
- 75 m

**Correct Answer:** Option d

**LOB:** Define radius, chord, diameter, segment (major and minor), arc (major and minor), interior or exterior of a circle in order to illustrate and label them on a given circle

1. Given below is a circle with center O.

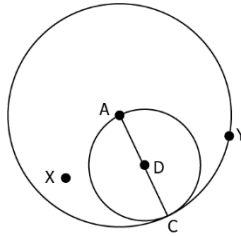


Which of the following is not true about the given figure?

- MGN is the minor sector
- $\widehat{MN}$  is the minor arc
- $\widehat{MDN}$  is the major arc
- AR is a chord

**Correct Answer:** Option a

2. Given below are two circles with center A and D.



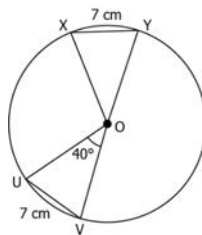
Which of the following is FALSE about the given figure?

- Point X lies inside the bigger circle.
- AC is a chord of the smaller circle.
- AC is the radius of the bigger circle.
- Smaller circle lies in the minor segment of the bigger circle.

**Correct Answer:** Option d

**LOB:** Apply theorems regarding angle subtended by a chord in a circle in order to find the measure of an angle in the given figure

1. Given below is a circle with centre O.

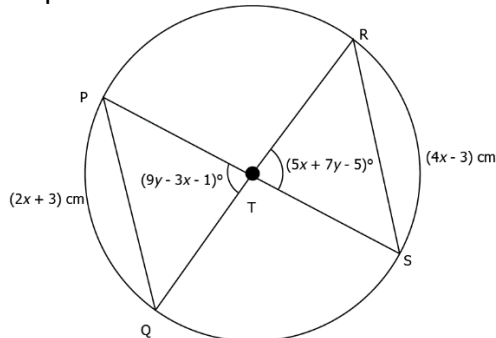


What is the measure of  $\angle XOY$ ?

- $40^\circ$
- $140^\circ$
- $80^\circ$
- $90^\circ$

**Correct Answer:** Option a

2. In the figure below, PQ and RS are equal chords of a circle with centre T.



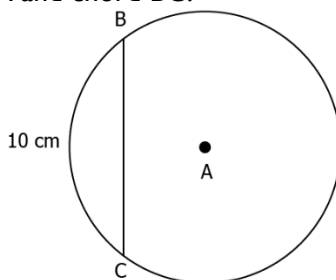
What is the measure of  $\angle PTQ$  ?

- $100^\circ$
- $40^\circ$
- $80^\circ$
- $60^\circ$

**Correct Answer:** Option c

**LOB:** Apply the property of perpendicular from the centre to the chord in order to solve for the missing values (lengths and angles) in a given figure

1. Given below is a circle with centre A and chord BC.

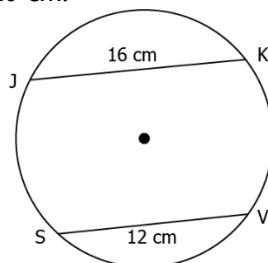


If radius of the chord is 13 cm, what is distance of the chord from the centre?

- 10 cm
- 13 cm
- 12 cm
- 5 cm

**Correct Answer:** Option c

2. Given below is a circle with diameter 20 cm.



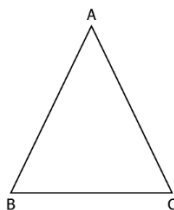
What is the distance between the chords? How much this distance will change if the chords are in the same direction of the centre?

- 14 cm; it will increase by 12 cm
- 7 cm; it will remain same
- 14 cm; it will decrease by 12 cm
- 2 cm; it will increase by 12 cm

**Correct Answer:** Option c

**LOB:** Construct circle passing through 1, 2 & 3 non-collinear points in order to comment on how many circles can be constructed passing through them

1. Which statement is not correct?



- Infinitely many circles can pass through all the points A, B and C.
- Infinitely many circles can pass through both the points A and B.
- Infinitely many circles can pass through the point A.
- Only two circles can pass through both the points B and C.

**Correct Answer:** Option a

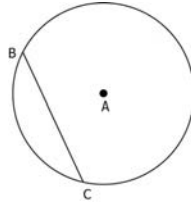
2. How many circle(s) can be constructed that passes through each vertex of  $\Delta PQR$ ?

- 4
- 3
- Infinitely many
- 1

**Correct Answer:** Option d

**LOB:** Use the value of radius and perpendicular to the chord in order to compute the length of a chord

1. Given below is a circle with centre A and radius 10 cm.

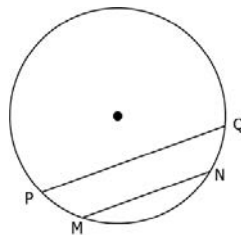


What is the length of BC if it is 8 cm away from the centre?

- 6 cm
- 12 cm
- 5 cm
- 4 cm

**Correct Answer:** Option b

2. Given below is a circle with radius 13 cm. PQ and MN are two chords of length 24 cm and  $x$  cm respectively. The distance between the chords is 7 cm. What is the value of  $x$ ?

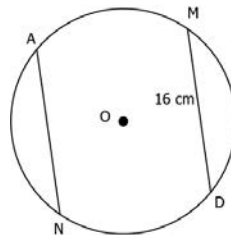


- 10 cm
- 5 cm
- 11 cm
- 6 cm

**Correct Answer:** Option a

**LOB:** Interpret and apply theorems on the angles subtended by arcs of a circle in order to solve for unknown values in given examples

1. In the figure below, AN and MD are equal chords of a circle with centre O and diameter 20 cm.

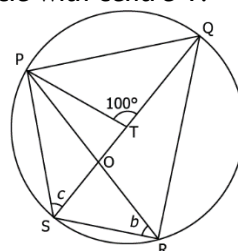


What is the distance between the two chords?

- 10 cm
- 12 cm
- 6 cm
- 25 cm

**Correct Answer:** Option b

2. Given below is a quadrilateral PQRS in a circle with centre T.



What are the measures of  $b$  and  $c$ ?

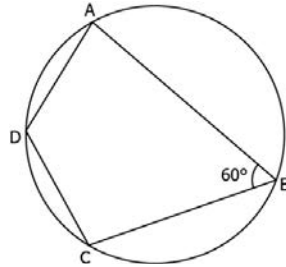
- $b = 50^\circ$  and  $c = 40^\circ$

- b.  $b = 80^\circ$  and  $c = 100^\circ$
- c.  $b = 100^\circ$  and  $c = 80^\circ$
- d.  $b = 40^\circ$  and  $c = 50^\circ$

**Correct Answer:** Option d

**LOB:** Apply the relation between angles of a cyclic quadrilateral in order to solve for the value of a given angle

1. In the figure below,  $\angle A : \angle C = 2:3$ .

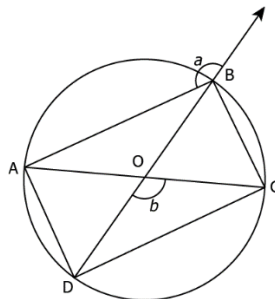


What are the measures of  $\angle C$  and  $\angle D$ ?

- a.  $\angle C = 120^\circ$  and  $\angle D = 108^\circ$
- b.  $\angle C = 108^\circ$  and  $\angle D = 120^\circ$
- c.  $\angle C = 120^\circ$  and  $\angle D = 108^\circ$
- d.  $\angle C = 72^\circ$  and  $\angle D = 120^\circ$

**Correct Answer:** Option b

2. Given below is a circle with centre O.



Which of these represents the measure of  $\angle BCD$ ?

- a.  $180^\circ + (a - \frac{b}{2})$
- b.  $180^\circ - (a + \frac{b}{2})$
- c.  $180^\circ - a - \frac{b}{2}$
- d.  $180^\circ - (a - \frac{b}{2})$

**Correct Answer:** Option d

# Suggested Teacher Resources



## Lesson Plan

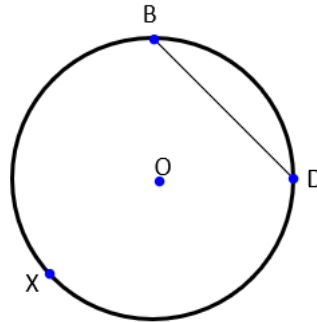


<b>Objectives</b>	To Define radius, chord, diameter, segment (major and minor), arc (major and minor), interior or exterior of a circle in order to illustrate and label them on a given circle
<b>Prerequisite Knowledge</b>	Basic knowledge of circles
<b>Material Required</b>	Paper cut outs of circles
<b>Procedure</b>	<p>The teacher will begin by distributing the paper circles to each student in the class. The teacher will begin by asking the students to name the shape of the paper cut out that they have been just given. <i>It is a circle.</i></p> <p>The teacher will then engage the students in a discussion given below:</p> <ul style="list-style-type: none"> <li>• What is a circle? The different responses by the students will be listed on the board. The response- 'The collection of all the points in a plane, which are at a fixed distance from a fixed point in the plane, is called a circle.' should be included.</li> <li>• Look at the outer edge of the circle. What is the boundary length of the circle called? <i>Circumference</i></li> <li>• Fold the circle directly in half and crease it well.</li> <li>• Open the circle, the crease that is made is the <i>diameter</i> of the circle.</li> <li>• Hold the circle at the ends of the crease. Fold the circle in half again, but this time ensure to match up the end points of the crease.</li> <li>• Open the circle, is this also a <i>diameter</i>? How do you know? Do the lines (intersect)? Yes. Is there something special about the way these lines intersect? They create four right angles. This special type of intersection is called perpendicular.</li> <li>• The point where both the diameter (creases) intersect is called the <i>center</i> of the circle. Place a dot at the point of intersection and name the center.</li> <li>• Now, trace one of the lines from the center to the edge of the circle. This line from the center is called the <i>radius</i>. Radius is equal to half the length of the diameter.</li> </ul> <div style="text-align: center;"> </div> <ul style="list-style-type: none"> <li>• In the above figure,             <ul style="list-style-type: none"> <li>○ O is the centre of the circle.</li> <li>○ AB and CD are the diameter.</li> <li>○ <math>AO = AB/2</math> (radius is equal to half the length of the diameter)</li> </ul> </li> </ul>

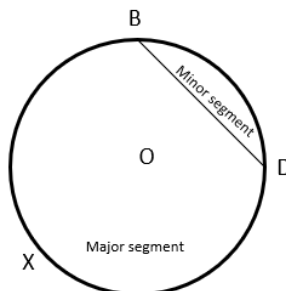


- $AO, OB, CO$  and  $OD$  are the radii of the circle which are equal in length. Plural of radius is radii.

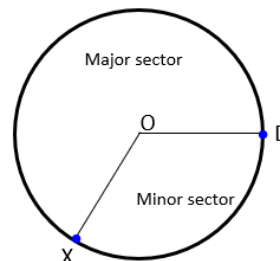
- Now, fold in one of the curved edges of the circle and crease it well.
- Open the fold and look at the crease that is just made. Is it a diameter? Is it a radius? Why or why not?  
This line is called a *chord*.
- Also, look at the curved part of the circle between the points where this line touches the outside of the circle. This is called an *arc*.
- Try finding other arcs on the circle. The arc which is longer is called *major arc* and the shorter arc is called the *minor arc*.



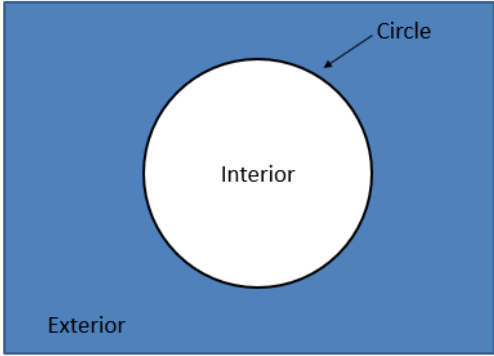
- In the above circle, with center  $O$ ,
  - $BD$  is a chord.
  - Arc  $BD$  is the minor arc and arc  $BXD$  is the major arc.
- Draw a segment connecting the arcs, the area enclosed by the chord connecting the major arc is called the *major segment* and the area enclosed by chord which connects to minor arc is called *minor segment*.



- Then, the teacher will introduce that the area enclosed by an arc and 2 radii are called *sectors*, the area enclosed by major arc and 2 radii is called *major sector* and the area enclosed by minor arc and 2 radii is called *minor sector*.



Then she will describe that a circle divides a plane in three parts:

	<div style="text-align: center;">  </div> <p>(i) Inside the circle, which is also called the interior of the circle.                  (ii) The circle                  (iii) Outside the circle, which is also called the exterior of the circle</p> <p>The circle and its interior make up the circular region.</p> <p>The teacher will then summarize the discussion on circle and its parts on the board.</p>
Reference	<a href="https://www.hofstra.edu/pdf/academics/colleges/soeahs/tll/matfl/matfl_circlegeometrylessonplan.pdf">https://www.hofstra.edu/pdf/academics/colleges/soeahs/tll/matfl/matfl_circlegeometrylessonplan.pdf</a>

2

## Activity



**Aim:** Verify that the angle subtended by an arc at the center of a circle is twice the angle subtended by the same arc at any other point on the remaining part of the circle.

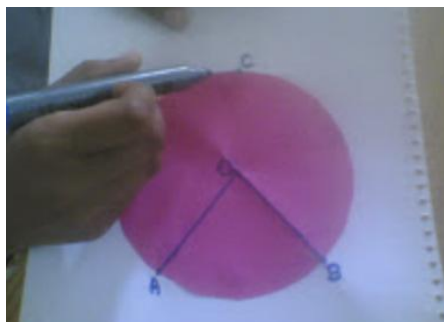
**Materials required:** Coloured paper, a pair of scissors, geometry box, glue, carbon paper.

**Procedure:**

1. Draw a circle of any radius with center O on a coloured paper and cut it.
2. Take a rectangular sheet of paper and paste the circle cut out on it.



3. Mark two points A and B on the circle to get arc AB.
4. Form a crease joining OA and draw OA.
5. Form a crease joining OB and draw OB.
6. Take a point C on the remaining part of the circle.



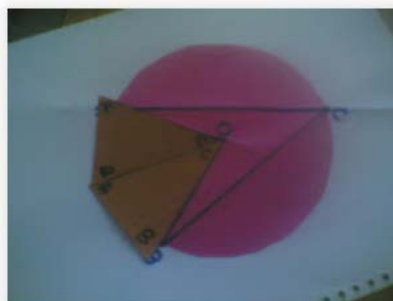
7. Form a crease joining AC and draw AC.



8. Form a crease joining BC and draw BC.  
9. Make two replicas of  $\angle ACB$  using carbon paper.



10. Place the two replicas of  $\angle ACB$  adjacent to each other on  $\angle AOB$ .



**Observations:**

1. Arc AB subtends  $\angle AOB$  at the center and  $\angle ACB$  at the point C on the remaining part of the circle.
2. Two replicas of  $\angle ACB$  completely cover  $\angle AOB$ .  
Result:  $\angle AOB = 2 (\angle ACB)$

**Reference:** [http://mykhmsmathclass.blogspot.com/2007/10/class-ix-activity-11\\_08.html?m=1](http://mykhmsmathclass.blogspot.com/2007/10/class-ix-activity-11_08.html?m=1)

# II. CONSTRUCTIONS

**QR Code:**



**Learning outcome and Learning Objectives:**

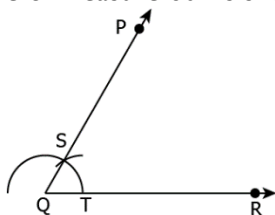
Content area/Concepts	Learning Objectives	Learning Outcome
Basic Constructions	List and execute steps of construction in order to bisect a given angle.	Constructs different geometrical shapes like bisectors of line segments, angles, and triangles under given conditions in order to provide reasons for the processes of such constructions
	List and execute steps of construction in order to draw the perpendicular bisector of a given line segment.	
	List and execute steps of construction in order to construct an angle of any given measurement	
Some Constructions of Triangles	List and execute steps of construction in order to construct a triangle given its base, a base angle and the sum of the other two sides.	
	List and execute steps of construction in order to construct a triangle given its base, a base angle and the difference of the other two sides	
	List and execute steps of construction in order to construct a triangle given its perimeter and its two base angles	

# Test items



**LOB:** List and execute steps of construction in order to bisect a given angle.

1. A part of construction to bisect an angle of measure  $60^\circ$  is shown below.

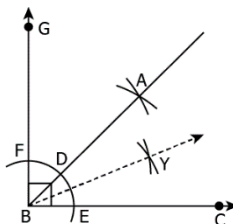


Which of these is the next step to complete the construction?

- With S and T as centres and with the radius more than  $\frac{1}{2}ST$ , draw arcs to intersect each other at U.
- With S and T as centres and with the radius less than  $\frac{1}{2}ST$ , draw arcs, say U and V.
- With S and T as centres and with the radius less than  $\frac{1}{2}ST$ , draw arcs to intersect each other at U.
- With only T as centre and with the radius less than  $\frac{1}{3}ST$ , draw arc, say U.

**Correct Answer:** Option a

2. Ankita bisected an angle of measure  $45^\circ$  as shown below:



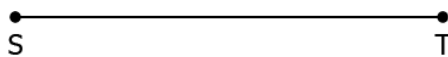
Which of these options shows that BY is the bisector of  $\angle ABC$ ?

- As  $\triangle BDY \cong \triangle BEY$  by SAS congruency criteria, and by CPCT  $\angle DYB = \angle EYB$ .
- As  $\triangle BDY \cong \triangle BEY$  by ASA congruency criteria, and by CPCT  $\angle DBY = \angle EBY$ .
- As  $\triangle BDY \cong \triangle BEY$  by SSS congruency criteria and by CPCT  $\angle DBY = \angle EBY$ .
- As  $\triangle BDY \cong \triangle BEY$  by SSS congruency criteria, and by CPCT  $\angle DYB = \angle EYB$ .

**Correct Answer:** Option c

**LOB:** List and execute steps of construction in order to draw the perpendicular bisector of a given line segment.

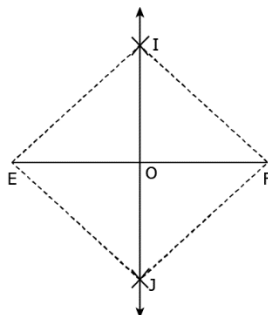
1. Which option describes the first two steps to construct the perpendicular bisector of ST?



- Taking S and T as centres and radius more than  $\frac{1}{2}ST$ , draw arcs on both sides of the line segment ST.  
Let these arcs intersect each other at P and Q and join PQ.
- Taking S and T as centres and radius less than  $\frac{1}{2}ST$ , draw arcs on both sides of the line segment ST.  
Let these arcs intersect each other at P and Q and join PS and PT.
- Taking S and T as centres and radius less than  $\frac{1}{3}ST$ , draw arcs on both sides of the line segment ST.  
Let these arcs intersect each other at P and Q and join PS and PT.
- Taking S and T as centres and radius less than  $\frac{1}{4}ST$ , draw arcs on both sides of the line segment ST.  
Let these arcs intersect each other at P and Q and join PQ.

**Correct Answer:** Option a

2. Observe the figure below.



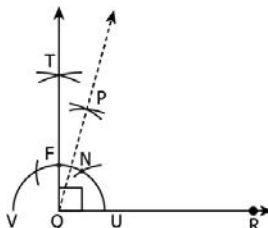
Which option shows the correct reasoning used to show that IOJ is the perpendicular bisector of EF?

- a.  $\triangle IEJ \cong \triangle IFJ$  by SAS rule
  - a.  $\triangle IOE \cong \triangle IOF$  by SSS rule
  - b.  $\angle IOE + \angle IOF = 90^\circ$
- b.  $\triangle IEJ \cong \triangle IFJ$  by SSS rule
  - $\triangle IOE \cong \triangle IOF$  by SAS rule
  - $\angle IOE + \angle IOF = 180^\circ$
- c.  $\triangle IEJ \cong \triangle IFJ$  by SSS rule
  - a.  $\triangle IOE \cong \triangle IOF$  by SSS rule
  - b.  $\angle IOE + \angle IOF = 90^\circ$
- d.  $\triangle IEJ \cong \triangle IFJ$  by SAS rule
  - a.  $\triangle IOE \cong \triangle IOF$  by SSS rule
  - b.  $\angle IOE + \angle IOF = 180^\circ$

**Correct Answer:** Option b

**LOB:** List and execute steps of construction in order to construct an angle of any given measurement

1. The construction of  $\angle PQR$  of measure  $75^\circ$  is shown below.

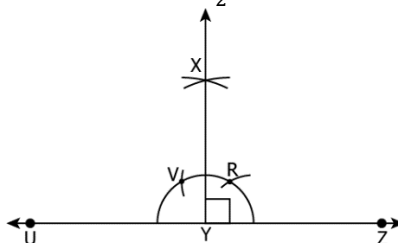


Which angle have been bisected to get  $\angle PQR$ ?

- a.  $\angle TQR$
- b.  $\angle TQN$
- c.  $\angle NQU$
- d.  $\angle TQP$

**Correct Answer:** Option b

2. Part of work to construct an angle of measure  $157\frac{1}{2}^\circ$  is shown below.



Which of the following are the next two steps?

- a. Bisect  $\angle XYU$ , to get  $\angle MYZ$  of measure  $135^\circ$ .  
Bisect  $\angle MYU$ , to get  $\angle NYZ$  of measure  $157\frac{1}{2}^\circ$ .
- b. Bisect  $\angle XYV$ , to get  $\angle TYZ$  of measure  $135^\circ$ .

- Bisect  $\angle TYU$ , to get  $\angle QYZ$  of measure  $157\frac{1}{2}^\circ$ .
- c. Bisect  $\angle XYU$ , to get  $\angle MYZ$  of measure  $105^\circ$ .  
Bisect  $\angle MYU$ , to get  $\angle NYZ$  of measure  $157\frac{1}{2}^\circ$ .
- d. Bisect  $\angle XYV$ , to get  $\angle TYZ$  of measure  $105^\circ$ .  
Bisect  $\angle TYU$ , to get  $\angle QYZ$  of measure  $157\frac{1}{2}^\circ$ .

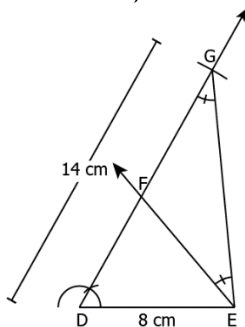
**Correct Answer:** Option a

**LOB:** List and execute steps of construction in order to construct a triangle given its base, a base angle and the sum of the other two sides.

1. Which number correctly completes the statement below?  
The construction of a triangle PQR, given that  $PQ = 18$  cm,  $\angle P = 60^\circ$  is possible when the sum of sides QR and PR is equal to \_\_.  
  - a. 12 cm
  - b. 16 cm
  - c. 18 cm
  - d. 22 cm

**Correct Answer:** Option d

2. Devika constructs a triangle DEF where  $DE = 8$  cm,  $\angle D = 60^\circ$  and  $DF + FE = 14$  cm.



Which of these options explains that she constructed the triangle DEF correctly?

- a. As  $\angle FGE = \angle FEG$  i.e.,  $FG = GE$  i.e.,  $DF = DG - FG = DG - GE$ , so,  $DF + GE = DG = 14$  cm.
- b. As  $\angle FGE = \angle FEG$  i.e.,  $FG = 2FE$ ,  $DF = DG - FG = DG - 2FE$ , so,  $DF + 2FE = DG = 14$  cm.
- c. As  $\angle FGE = \angle FEG$  i.e.,  $FG = FE$ ,  $DF = DG - FG = DG - FE$ , so,  $DF + FE = DG = 14$  cm.
- d. As  $\angle FGE = \angle FEG$  i.e.,  $FG = 2GE$  i.e.,  $DF = DG - FG = DG - 2GE$ , so,  $DF + 2GE = DG = 14$  cm.

**Correct Answer:** Option c

**LOB:** List and execute steps of construction in order to construct a triangle given its base, a base angle and the difference of the other two sides

1. Arjun performed the following steps to construct a triangle MNO, where  $MN = 8$  cm,  $\angle M = 45^\circ$  and  $NO - MO = 0.7$ .

**Step 1:** Draw the base MN and at point M make an angle say  $\angle XMN = 45^\circ$ .

**Step 2:** Cut line segment MK equal to  $NO - MO$  from the line MX.

**Step 3:** Join TN and draw perpendicular bisector, say IJ of TN.

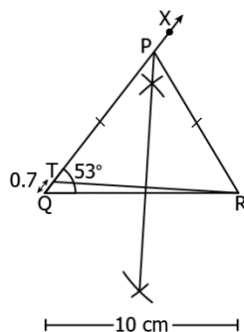
**Step 4:** Let IJ intersect MX at O and then join ON.

If he made an error in the steps of construction, in which step did Arjun make the first error?

- a. Step 1
- b. Step 2
- c. Step 3
- d. Step 4

**Correct Answer:** Option b

2. Part of the work to construct a triangle PQR, where  $QR = 10$  cm,  $\angle Q = 53^\circ$  and  $PQ - PR = 0.7$  as shown below.



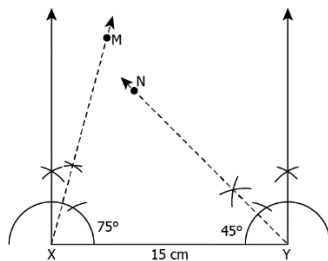
Which of the following options explains that triangle PQR is constructed correctly?

- a. As,  $PT = PR$ . So,  $QT = PT - PQ = PT - PQ$
- b. As,  $PT = PR$ . So,  $QT = PQ - PT = PQ - PR$
- c. As,  $PT = PR$ . So,  $2QT = PT - PQ = PT - PQ$
- d. As,  $PT = PR$ . So,  $2QT = PQ - PT = PQ - PR$

**Correct Answer:** Option b

**LOB:** List and execute steps of construction in order to construct a triangle given its perimeter and its two base angles

1. A part of work to construct a triangle JKL, where  $\angle J = 75^\circ$ ,  $\angle K = 45^\circ$  and  $JK + KL + JL = 15$  cm is shown below:

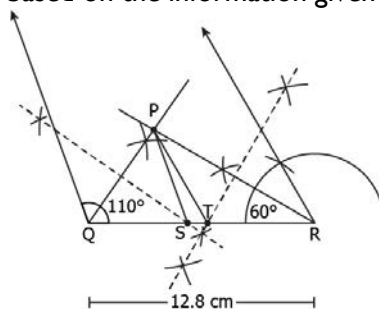


Which of these options shows the next step of construction?

- a. Bisect  $\angle MXY$  and  $\angle NYX$  and let these bisectors intersect at point J.
- b. Draw perpendicular bisectors of MX and NY.
- c. Bisect  $\angle MXY$  and draw perpendicular bisector of NY.
- d. Bisect  $\angle NYX$  and draw perpendicular bisector of MX.

**Correct Answer:** Option a

2. Bhavna constructed a triangle PST based on the information given to her.



What information was provided to her?

- a. The perimeter of the triangle is 12.8 cm and the two base angles  $\angle S$  and  $\angle T$  are  $60^\circ$  and  $110^\circ$  respectively.
- b. The perimeter of the triangle is 12.8 cm and the two base angles  $\angle S$  and  $\angle T$  are  $110^\circ$  and  $60^\circ$  respectively.
- c. The perimeter of the triangle is 6.4 cm and the two base angles  $\angle S$  and  $\angle T$  are  $110^\circ$  and  $60^\circ$  respectively.
- d. The perimeter of the triangle is 6.4 cm and the two base angles  $\angle S$  and  $\angle T$  are  $60^\circ$  and  $110^\circ$  respectively.

**Correct Answer:** Option b



# Suggested Teacher Resources



## Lesson Plan



Objectives	Students will be able to bisect angles using a piece of paper
Prerequisite Knowledge	<ul style="list-style-type: none"> <li>• Meaning of Angle bisector.</li> <li>• How to calculate half of a given angle.</li> </ul>
Material Required	Compass, protractor and scale.
Procedure	<ol style="list-style-type: none"> <li>1. Teacher will start the class by asking various properties of triangles.</li> <li>2. Teacher will then divide the class into 4 groups (A,B,C,D) and she will prepare 3 chits for the class.</li> <li>3. On each chit, following conditions of construction will be written: <ul style="list-style-type: none"> <li>• To construct a triangle given its base, a base angle and the sum of the other two sides.</li> <li>• To construct a triangle given its base, a base angle and the difference of the other two Sides.</li> <li>• To construct a triangle given its perimeter and its two base angles.</li> </ul> </li> <li>4. For the first round, group A will pick a chit and read the condition of construction.</li> <li>5. After reading the condition, other groups will give group A one-one measurement for the construction. For example, if group A picks a chit: To construct a triangle given its base, a base angle and the sum of the other two sides, group B will give base dimension, group C will give base angle and group D will give sum of the other two sides. Group A will take the measurements and construct the required triangle.</li> <li>6. Similarly, for the next round, group B will pick the chit and other groups will tell the dimensions.</li> <li>7. Activity will continue in this fashion till each team constructs 2-3 triangles.</li> <li>8. Lesson will be concluded by asking students to verify properties of triangles through following questions with the students: <ul style="list-style-type: none"> <li>• The sum of all the angles of a triangle (of all types) is equal to 180°.</li> <li>• The sum of the length of the two sides of a triangle is greater than the length of the third side.</li> <li>• The difference between the two sides of a triangle is less than the length of the third side.</li> <li>• The side opposite the greater angle is the longest side of all the three sides of a triangle.</li> <li>• The exterior angle of a triangle is always equal to the sum of the interior opposite angles.</li> </ul> </li> </ol>



**OBJECTIVE:** To verify experimentally that if two lines intersect each other, then

1. the vertically opposite angles are equal.
2. the sum of two adjacent angles is  $180^\circ$ .
3. the sum of all the four angles is  $360^\circ$ .

**Materials Required**

1. Cardboard
2. White paper
3. A full protractor
4. A nail
5. Two transparent strips marked as AB and CD
6. Adhesive

**Prerequisite Knowledge**

1. Basic knowledge of lines and angles.
2. Pair of angles; adjacent angles, linear pair of angles, vertically opposite angles.

**Theory**

**I. Lines and Angles**

1. **Line Segment:** A part of a line with two endpoints, is called a line segment. Line segment AB is denoted by  $\overline{AB}$ .

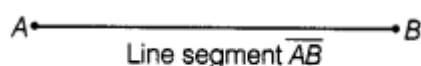


Fig. 11.1

A line segment has a definite length, which can be measured. The line segment  $\overline{AB}$  is the same thing as the line segment  $\overline{BA}$ .

2. **Angle:** The figure formed by two rays with the same initial point, is called an angle.

In Fig. 11.2, the common initial point B is known as the vertex of the angle and the rays ( $\overrightarrow{BA}$  and  $\overrightarrow{BC}$ ) forming the angle are called its arms or sides.

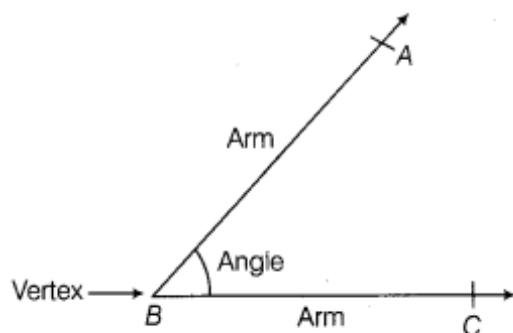


Fig. 11.2

There are different types of angles such as acute angle, right angle, obtuse angle, straight angle, reflex angle and complete angle, which are discussed below :

1. **(a) Acute Angle:** An angle whose measure is more than  $0^\circ$  but less than  $90^\circ$ , is called an acute angle.

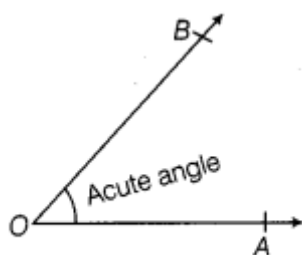


Fig. 11.3

In Fig. 11.3,  $\angle AOB$  is an acute angle.  
 Since,  $0^\circ < \angle AOB < 90^\circ$

2. **(b) Right Angle:** An angle whose measure is  $90^\circ$ , is called a right angle.

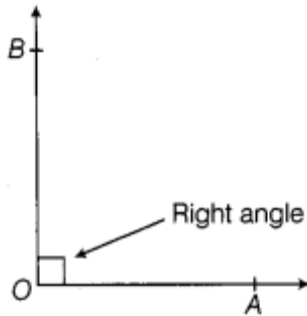


Fig. 11.4

In Fig. 11.4,  $\angle AOB$  is a right angle and  $BO \perp OA$ .

3. **(c) Obtuse Angle:** An angle whose measure is more than  $90^\circ$  but less than  $180^\circ$ , is called an obtuse angle.

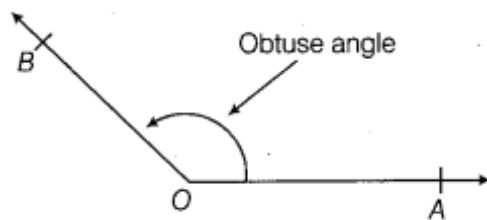


Fig. 11.5

In Fig. 11.5,  $\angle AOB$  is an obtuse angle.

Since,  $90^\circ < \angle AOB < 180^\circ$

4. **(d) Straight Angle:** An angle whose measure is  $180^\circ$ , is called a straight angle. In Fig. 11.6,  $\angle AOB = 180^\circ$  is a straight angle.

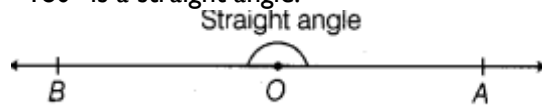


Fig. 11.6

A straight angle has two right angles.

5. **(e) Reflex Angle:** An angle whose measure is more than  $180^\circ$  but less than  $360^\circ$ , is called a reflex angle.

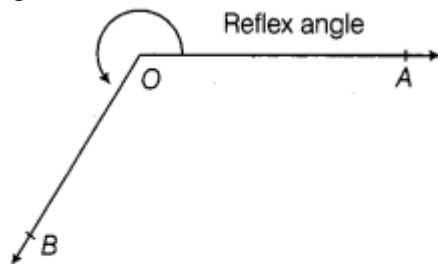


Fig. 11.7

In Fig. 11.7 and Fig. 11.8,  $\angle AOB$  and  $\angle PQR$  are reflex angles.

$180^\circ < \text{reflex } \angle AOB < 360^\circ$

$180^\circ < \text{reflex } \angle PQR < 360^\circ$

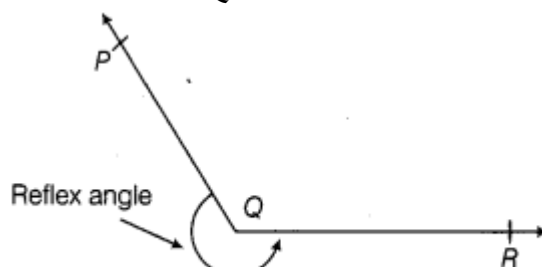


Fig. 11.8

6. **(f) Complete Angle:** An angle whose measure is  $360^\circ$ , is called a complete angle. In Fig. 11.9,  $\angle AOA = 360^\circ$  is a complete angle.



Fig. 11.9

## 2. Pair of Angles

There are some relations between the angles which are described below:

1. **Adjacent Angles:** Two angles are called adjacent angles, if

1. **(a)** they have a common vertex,
2. **(b)** they have a common arm and
3. **(c)** their non-common arms are on different sides of the common arm.

In Fig. 11.10,  $\angle AOC$  and  $\angle COB$  are adjacent angles because these angles have a common vertex  $O$ , a common arm  $OC$  and non-common arms  $OA$  and  $OB$  are on different sides of the common ray  $OC$ .

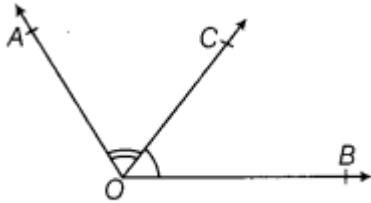


Fig. 11.10

When two angles are adjacent, then their sum is always equal to the angle formed by the two non-common arms. So, here  $\angle AOB = \angle AOC + \angle COB$ .

Note Here,  $\angle AOB$  and  $\angle AOC$  are not adjacent angles because their non-common arms  $OC$  and  $OB$  lie on the same side of the common arm  $OA$

2. **Linear Pair of Angles:** If the non-common arms of two adjacent angles form a line, then these angles are called linear pair of angles.

In Fig. 11.11,  $\angle AOC$  and  $\angle BOC$  form a linear pair of angles.

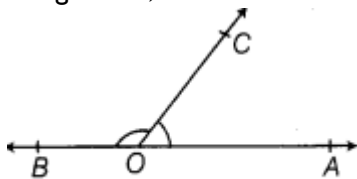


Fig. 11.11

3. **Vertically Opposite Angles:** Two angles are called a pair of vertically opposite angles, if their arms form two pairs of opposite rays.

In other words, when two lines intersect each other at a point, then there are two pairs of vertically opposite angles.

In Fig. 11.12, lines  $AB$  and  $CD$  intersect each other at  $O$ . So,  $\angle AOC$  and  $\angle BOD$  are vertically opposite angles. Also,  $\angle AOD$  and  $\angle BOC$  are vertically opposite angles.

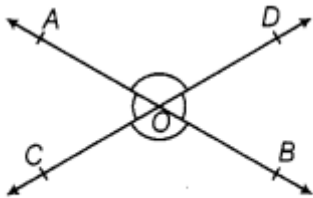


Fig. 11.12

### Procedure

1. Take a cardboard of suitable size and by using adhesive paste a white paper on it.
2. Also, paste a full protractor ( $0^\circ$  to  $360^\circ$ ) with the help of adhesive on the cardboard. (see Fig. 11.13)

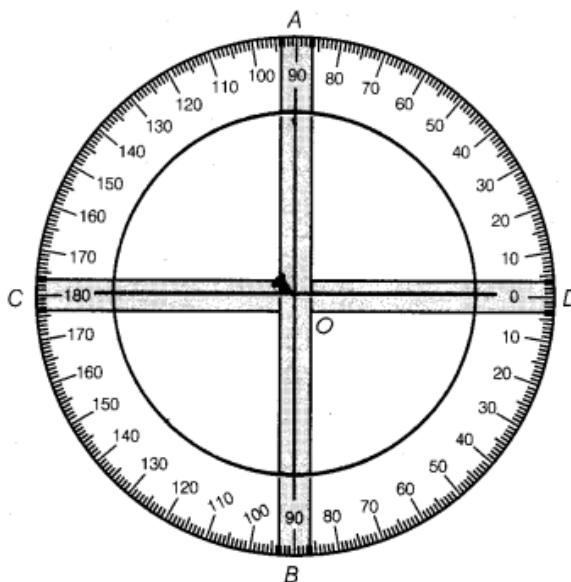


Fig. 11.13

3. Mark the centre of protractor as O.
4. Now, make a hole in the middle of both transparent strips which contain two intersecting lines.
5. Fix both strips at O by putting a nail, (see Fig. 11.13)

### Demonstration

In the different positions of the strips, observe the adjacent angles and the vertically opposite angles.

In the different positions, also compare vertically opposite angles formed by the two lines.

Check the relationship between the vertically opposite angles.

Check whether the vertically opposite angles,  $\angle AOD$  and  $\angle COB$  are equal.

Similarly, check whether the vertically opposite angles,  $\angle BOD$  and  $\angle AOC$  are equal.

Find the sum of two adjacent angles such that  $\angle AOD + \angle AOC$  which is equal to  $180^\circ$ .

$$\text{i.e. } \angle AOC + \angle COB = \angle COB + \angle BOD$$

$$= \angle BOD + \angle AOD = 180^\circ$$

Now, we obtain the sum of all the four angles formed at the point O and it is equal to  $360^\circ$ .

### Observation

In one position of the strips, by actual measurement of angles

1.  $\angle AOD = \dots\dots\dots$ ,  $\angle AOC = \dots\dots\dots$ ,  
 $\angle COB = \dots\dots\dots$ ,  $\angle BOD = \dots\dots\dots$ ,  
 Hence,  $\angle AOD = \angle COB$  and  $\angle AOC = \angle BOD$  (vertically opposite angles)
2.  $\angle AOC + \angle AOD = \dots\dots\dots$ ,  $\angle AOC + \angle BOC = \dots\dots\dots$ ,  
 $\angle COB + \angle BOD = \dots\dots\dots$ ,  $\angle AOD + \angle BOD = \dots\dots\dots$  (linear pairs)
3.  $\angle AOD + \angle AOC + \angle COB + \angle BOD = \dots\dots\dots$  (angles formed at a point)

### Result

We have verified experimentally that if two lines intersect each other, then

1. the vertically opposite angles are equal.
2. the sum of two adjacent angles is  $180^\circ$ .
3. the sum of all the four angles is  $360^\circ$ .

### Application

These properties are very useful in several geometrical operations.

# 12. HERON'S FORMULA

**QR Code:**



**Learning outcome and Learning Objectives:**

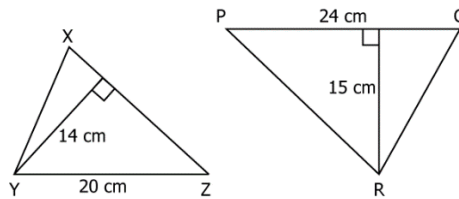
Content area/Concepts	Learning Objectives	Learning Outcome
Introduction	Calculate area of a given triangle to state the limitation of the Standard formula (Area of Triangle = $1/2 \times b \times h$ )	Applies appropriate formulae in order to find areas of all types of triangles
Area of a Triangle by Heron's formula	Apply Heron's formula in order to calculate the area of a Triangle	
Application of Heron's Formula in finding Areas of Quadrilateral	Breakdown a given polygon into triangles in order to find the area of a given polygon as a sum of areas of those triangles	

# Test items



**LOB:** Calculate area of a given triangle to state the limitation of the Standard formula (Area of Triangle =  $1/2 \times b \times h$ )

1. Two triangles are shown below.

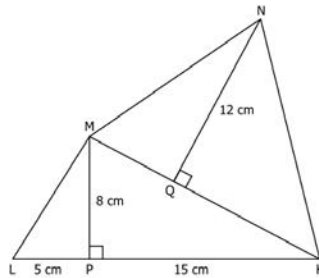


Which of following is true?

- Area of both the triangles can be calculated, area of  $\triangle XYZ = 140 \text{ cm}^2$  and area of  $\triangle PQR = 180 \text{ cm}^2$
- Area of only triangle XYZ can be calculated, area of  $\triangle XYZ = 140 \text{ cm}^2$
- Area of only triangle PQR can be calculated, area of  $\triangle PQR = 180 \text{ cm}^2$
- Area of both the triangles cannot be calculated

**Correct Answer:** Option c

2. A figure is shown below.



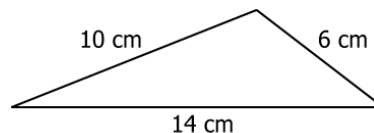
Can we find the area of the quadrilateral KLMN?

- No, as the side MK is unknown.
- No, as the side NK is unknown.
- Yes, and the area of the quadrilateral KLMN is  $192 \text{ cm}^2$ .
- Yes, and the area of the quadrilateral KLMN is  $384 \text{ cm}^2$ .

**Correct Answer:** Option c

**LOB:** Apply Heron's formula in order to calculate the area of a Triangle

1. A triangle is shown below.

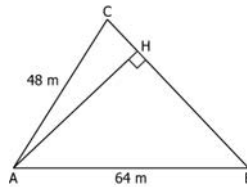


Which of following is equal to the area of the triangle?

- $\sqrt{(9)(5)(1)}$
- $\sqrt{15(9)(5)(1)}$
- $\sqrt{(24)(20)(16)}$
- $\sqrt{30(24)(20)(16)}$

**Correct Answer:** Option b

2. A triangle is shown below.



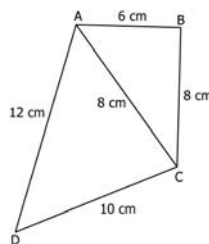
If the perimeter of the triangle is 192m, what is the length of AH?

- a. 38.4 m
- b. 40 m
- c. 76.8 m
- d. 80 m

**Correct Answer:** Option a

**LOB:** Breakdown a given polygon into triangles in order to find the area of a given polygon as a sum of areas of those triangles

1. A quadrilateral ABCD is shown below.

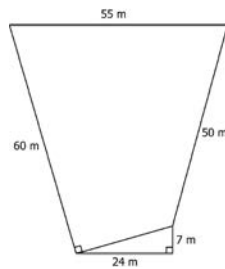


Which is the area of ABCD?

- a. 61.94 cm<sup>2</sup>
- b. 86.96 cm<sup>2</sup>
- c. 123.88 cm<sup>2</sup>
- d. 173.92 cm<sup>2</sup>

**Correct Answer:** Option a

2. The sketch of a farm is shown below.



What is the area of the farm?

- a. 1336.04 m<sup>2</sup>
- b. 2140.04 m<sup>2</sup>
- c. 2170.04 m<sup>2</sup>
- d. 3004.04 m<sup>2</sup>

**Correct Answer:** Option c



# Suggested Teacher Resources



## Lesson Plan



Objectives	Students will verify the area of triangles using heron's formula.
Prerequisite Knowledge	Heron's formula, Working on geoboard
Material Required	Geoboard, Rubber Bands.
Procedure	<ol style="list-style-type: none"> <li>1. Teacher will divide the class into groups of 4.</li> <li>2. Each group will be given a geoboard and some rubber bands.</li> <li>3. Teacher will ask the groups to make triangles of their own choice and measure their sides with a scale.</li> <li>4. Students will be instructed to replicate those triangles on graph paper and calculate the area by counting the squares.</li> <li>5. Students will be asked to verify the area of those triangles through Heron's formula as well.</li> <li>6. Then teacher will ask the students to solve the following questions through heron's formula: <ul style="list-style-type: none"> <li>• Find the area of a right triangle with one of the sides and the hypotenuse as 3 and 5cm respectively.</li> <li>• Find the area of an equilateral triangle with a side 10cm long.</li> <li>• Express the altitude of an isosceles triangle with two equal sides 'a' and base 'b' in terms of a and b.</li> <li>• Evaluate the area of the triangle whose perimeter is 180cm and two of its sides are 80cm and 18cm.</li> </ul> </li> </ol>

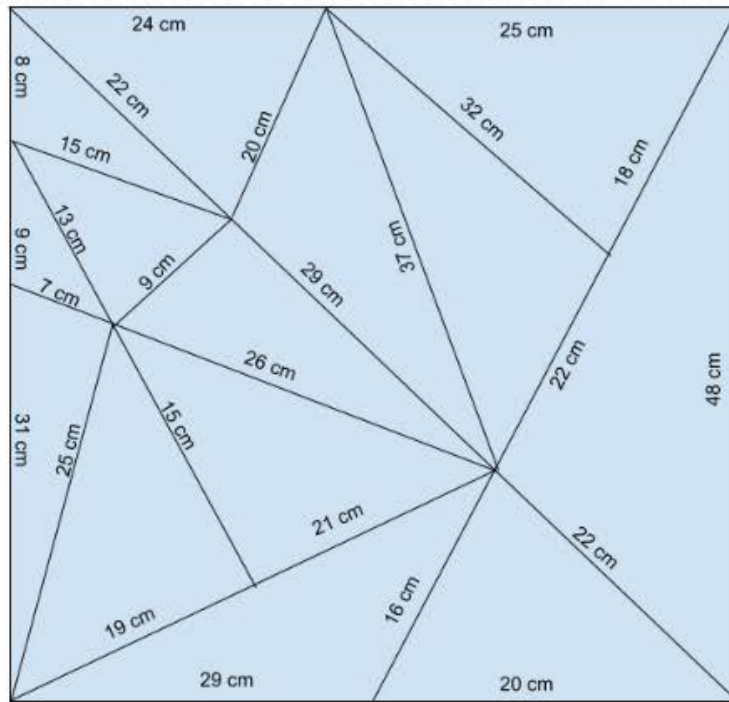


## Activity



**Setup:** Draw this figure on the board.

Students in groups of 4 will find the area of the triangles using Heron's formula and shade/colour the triangles with the same area.



3

## Activity



Objective: students will be able to calculate the area of triangles using Heron's formula.

Materials

- Popsicle sticks
- Marker
- Paper
- Pencils

Teacher Directions

Preparation

Prior to the game, write various numbers on popsicle sticks.

Game

1. Show students Heron's formula and discuss how to use it to find the area of a triangle.
2. Model using the formula with several example triangles.
3. Divide the class into pairs, and provide each pair with a handful of the popsicle sticks you created, paper, and pencils. Each pair should have an equal number of popsicle sticks.
4. When you say 'go,' students will group their popsicle sticks into groups of three and create triangles.
5. Students will use the numbers on each side of the triangles they created as the measurements of the sides. Using these numbers, students will calculate the area of each triangle using Heron's formula and write the answer on their paper.
6. The first team to correctly use Heron's formula to calculate the areas of the triangles they created wins.

Discussion Questions

- Why do you need to find 's' before you can complete Heron's formula?
- How is Heron's formula different from the formula: area of a triangle = (base x height)/2? How can they both be right?

Source: <https://study.com/academy/lesson/herons-formula-games-activities.html>

# 13. SURFACE AREAS AND VOLUMES

**QR Code:**



**Learning outcome and Learning Objectives:**

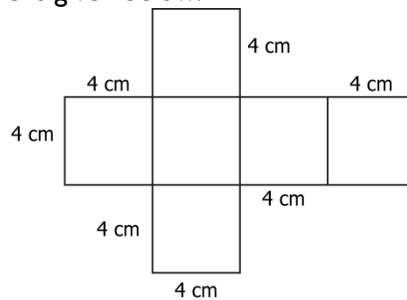
Content area/Concepts	Learning Objectives	Learning Outcome
Surface Area of a Cuboid and a Cube	Visualise a cube and cuboid in its 2-D form in order to calculate the surface area	Derives formulas for surface areas and volumes of different solid objects like, cubes, cuboids, right circular cylinders/ cones, spheres and hemispheres in order to apply them to objects found in the surroundings
	Calculate the surface area (lateral and total) of the cube or cuboid in order to determine the cost of painting/covering the given surface	
Surface Area of a Right Circular Cylinder	Visualize a cylinder in its 2-D form in order to calculate the curved surface area and total surface area	
	Calculate the surface area (curved and total) of a cylinder to determine the cost of painting/covering the given surface	
Surface Area of a Right Circular Cone	Visualize a right circular cone in 2-D in order to calculate the surface area (curved and total)	
	Calculate the surface area (curved and total) of a cone to determine the cost of painting/covering the given surface	
Surface Area of a Sphere	Calculate the surface area of a sphere/hemisphere to determine the cost of painting/covering the given surface of a sphere/hemisphere	
Volume of a Cube	Calculate the volume of a given cube in order to infer the quantity of any substance it can hold	
Volume of a Cuboid	Calculate the volume of a given cuboid in order to infer the quantity of any substance it can hold	
Volume of a Cylinder	Calculate the volume of a given cylinder in order to infer the quantity of any substance it can hold	
Volume of a Cone	Calculate the volume of a given cone in order to infer the quantity of any substance it can hold	
Volume of a sphere	Calculate the volume of a given sphere in order to infer the quantity of any substance it can hold	
Volume of a hemisphere	Calculate the volume of a given hemisphere in order to infer the quantity of any substance it can hold	

# Test items



**LOB:** Visualize a cube and cuboid in its 2-D form in order to calculate the surface area

1. The 2-D representation of a figure is given below:



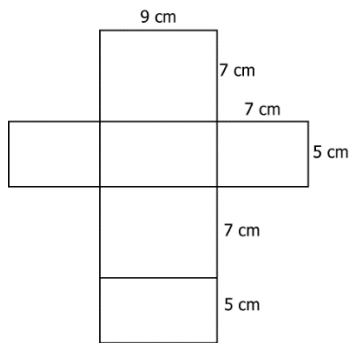
Which of these expressions represents the total surface area of the figure?

- a.  $[4^2] \text{ cm}^2$
- b.  $[4^2 + 4^2 + 4^2 + 4^2] \text{ cm}^2$
- c.  $[4^2 + 4^2 + 4^2 + 4^2 + 4^2] \text{ cm}^2$
- d.  $[4^2 + 4^2 + 4^2 + 4^2 + 4^2 + 4^2] \text{ cm}^2$

**Correct Answer:** Option d

2. Which of these is a 2-D representation of the cuboid whose total surface area is 510 cm<sup>2</sup>?

- a.
- 
- b.
- 
- c.
-



d.

**Correct Answer:** Option a

**LOB:** Calculate the surface area (lateral and total) of the cube or cuboid in order to determine the cost of painting/covering the given surface

1. Ajay needs to cover 15 cube shaped boxes each of side length 10 cm using a paper. If 1 cm<sup>2</sup> of paper costs him ₹0.25, what is the total cost of covering 15 boxes?
  - a. ₹1,125
  - b. ₹1,500
  - c. ₹2,250
  - d. ₹3,750

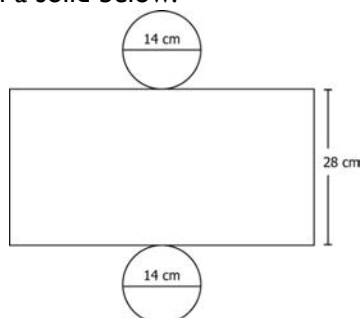
**Correct Answer:** Option c

2. Rajeev is painting 18 identical boxes. He paints the top and bottom with red colour and the remaining four faces with blue colour. He paints a total of 3.1680 m<sup>2</sup> area with blue color. If each box is 32 cm long and 22 cm deep, what is the breadth of each box?
  - a. 3.25 cm
  - b. 8 cm
  - c. 22.5 cm
  - d. 45 cm

**Correct Answer:** Option b

**LOB:** Visualize a cylinder in its 2-D form in order to calculate the curved surface area and total surface area

1. Consider the 2-D representation of a solid below:

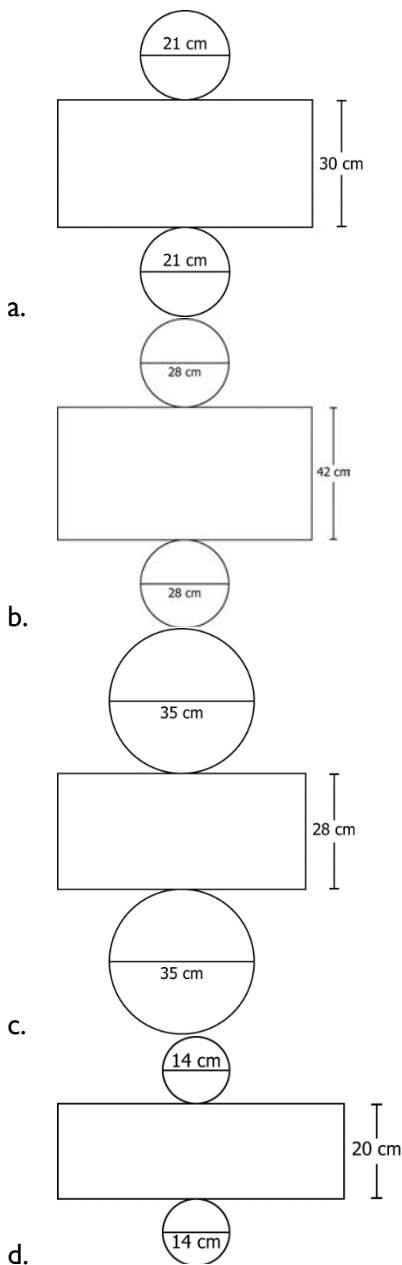


Which of these options represents the curved surface of the given solid? (Use  $\frac{22}{7}$  for  $\pi$ )

- a.  $2 \times \pi \times 14 \times 28 = 2,464\text{cm}^2$
- b.  $2 \times \pi \times 7 \times 28 = 1,232\text{cm}^2$
- c.  $2 \times \pi \times 14(14 + 28) = 3,696\text{cm}^2$
- d.  $2 \times \pi \times 7(7 + 28) = 1,540\text{cm}^2$

**Correct Answer:** Option b

2. Which of these represents the net of a cylinder whose lateral surface area is 880 cm<sup>2</sup> and total surface area is 1,188 cm<sup>2</sup>? (Use  $\frac{22}{7}$  for  $\pi$ )



**Correct Answer:** Option d

**LOB:** Calculate the surface area (curved and total) of a cylinder to determine the cost of painting/covering the given surface

1. Jatin made a model in the shape of a cylinder of radius 7 cm and height 14 cm for his school project. He wants to use colored sheet to decorate the model except the top and bottom. If the coloured sheet costs ₹0.75 per cm<sup>2</sup>, what will be the total cost to decorate the model? (Use  $\frac{22}{7}$  for  $\pi$ )
  - a. ₹462
  - b. ₹693
  - c. ₹924
  - d. ₹1,617

**Correct Answer:** Option a

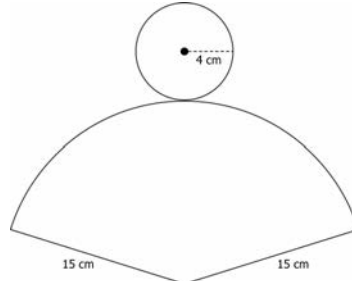
2. A factory manufactures cylindrical storage tanks made of steel whose top and bottom ends are closed in batches. The curved surface area and height of each tank is 22000 cm<sup>2</sup> and 100 cm respectively. If each batch contains a dozen of storage tanks and the cost of steel sheets used in manufacturing is ₹18 per square centimeter, how much would it cost to manufacture a batch?
  - a. ₹ 3,56,400
  - b. ₹ 5,34,600
  - c. ₹ 47,52,000

d. ₹ 64,15,200

**Correct Answer:** Option d

09M1305: Visualize a right circular cone in 2-D in order to calculate the surface area (curved and total)

1. The 2-D representation of a figure is given below.



Which of these expressions represent the curved surface area of the figure?

- a.  $\pi \times 4 \text{ cm} \times 15 \text{ cm}$
- b.  $\pi \times (4 \text{ cm} + 15 \text{ cm})$
- c.  $\pi \times 4 \text{ cm} \times (4 \text{ cm} + 15 \text{ cm})$
- d.  $2 \times \pi \times 4 \text{ cm} \times (4 \text{ cm} + 15 \text{ cm})$

**Correct Answer:** Option a

2. Which of these represents the correct net of the right circular cone whose total surface area is 301.44 feet<sup>2</sup>?

- a.
- b.
- c.
- d.

**Correct Answer:** Option d

**LOB:** Calculate the surface area (curved and total) of a cone to determine the cost of painting/covering the given surface

1. A heap of wheat is in the form of a cone whose diameter is 4.2 m and height is 2.8 m. The heap is to be covered exactly by a canvas to protect it from rain. If the rate of the canvas is ₹3 per m<sup>2</sup>, what is the total cost of the canvas needed to cover the heap?
  - a. ₹69.3
  - b. ₹82.53
  - c. ₹144.3
  - d. ₹201.96

**Correct Answer:** Option a

2. Asif is building a birdhouse. The roof of the birdhouse is in the form of a right circular cone whose radius is 21 cm and slant height is 47 cm. He plans to paint the roof red, but he needs to know the surface area to buy the right amount of paint. What is the surface area of the roof, including the bottom?
  - a.

**Correct Answer:** Option a

**LOB:** Calculate the surface area of a sphere/hemisphere to determine the cost of painting/covering the given surface of a sphere/hemisphere

1. A hemispherical bowl made of brass has inner radius 5.25 cm. What is the total cost of tin-plating it on the inside at the rate of ₹4 per cm<sup>2</sup>?
  - a. ₹6.93
  - b. ₹69.3
  - c. ₹693
  - d. ₹6930

**Correct Answer:** Option c

2. A hemispherical dome of a tomb needs to be painted. The circumference of the base of the dome is 17.6 cm. If the cost of painting is ₹7 per cm<sup>2</sup>, what is the cost, rounded to the nearest rupee, to paint the dome? (Use  $\pi$  as  $\frac{22}{7}$ )
  - a. ₹345
  - b. ₹390
  - c. ₹571
  - d. ₹690

**Correct Answer:** Option a

**LOB:** Calculate the volume of a given cube in order to infer the quantity of any substance it can hold

1. A milk tank is in the form of cube whose edge length is 8 m. How much quantity of milk, in cubic metres, can be stored in the tank?
  - a. 64
  - b. 384
  - c. 512
  - d. 800

**Correct Answer:** Option c

2. The capacity of a warehouse is usually measured by 12 m × 12 m × 12 m. What is the maximum number of cartons each measuring 0.75 m × 0.75 m × 0.75 m that can be stored in the warehouse?
  - a. 512
  - b. 987
  - c. 1727
  - d. 4096

**Correct Answer:** Option d

**LOB:** Calculate the volume of a given cuboid in order to infer the quantity of any substance it can hold

1. A tank in the shape of cuboid is 45 cm long, 25 cm wide and 35 cm deep. What is the capacity, in cubic centimeters, of the tank?
  - a. 102



- b. 3,575
- c. 5,150
- d. 39,375

**Correct Answer:** Option d

2. Abhinav stored 1,750 cuboidal cartons each measuring 30 cm × 45 cm × 20 cm in a hall, which is completely filled. The hall is 4.5 m long and 3.5 m wide, what is the height of the hall?
- a. 300 cm
  - b. 200 cm
  - c. 160 cm
  - d. 100 cm

**Correct Answer:** Option a

**LOB:** Calculate the volume of a given cylinder in order to infer the quantity of any substance it can hold

1. A cylindrical water storage tank has an inside base radius of 7 m and depth of 11 m. How many cubic centimeters of water can it hold? (Use  $\pi$  as  $\frac{22}{7}$ )
- a. 3388 m<sup>3</sup>
  - b. 1694m<sup>3</sup>
  - c. 847 m<sup>3</sup>
  - d. 484 m<sup>3</sup>

**Correct Answer:** Option b

2. After painting his car parking, Ravi has  $\frac{1}{5}$  of a cylindrical can of paint remaining. The can has a radius of 15 cm and a height 30 cm. He wants to pour the remaining paint into a smaller can for storage. The smaller can has a radius of 10 cm. What should be the minimum height of the smaller can in order to hold all of the paint?
- a. 67.5 cm
  - b. 45 cm
  - c. 13.5 cm
  - d. 4:9 cm

**Correct Answer:** Option c

**LOB:** Calculate the volume of a given cone in order to infer the quantity of any substance it can hold

1. How many cubic centimeters of water can a conical vessel of base diameter 42 cm and slant height 29 cm hold?
- a. 1,914 cm<sup>3</sup>
  - b. 9,240 cm<sup>3</sup>
  - c. 13,398cm<sup>3</sup>
  - d. 36,960cm<sup>3</sup>

**Correct Answer:** Option b

2. The radius and height of a conical cup is in the ratio 3:4. If the volume of the cup is 2,376 cm<sup>3</sup>, which of these can be slant height of the cone, rounded off to the nearest whole number?
- a. 3 cm
  - b. 4 cm
  - c. 15 cm
  - d. 20 cm

**Correct Answer:** Option d

**LOB:** Calculate the volume of a given sphere in order to infer the quantity of any substance it can hold

1. What is the volume, rounded to the nearest whole number, of metallic spherical ball of radius 4.5 cm? (Use  $\pi$  as  $\frac{22}{7}$ )
- a. 19 cm<sup>3</sup>
  - b. 38cm<sup>3</sup>
  - c. 156 cm<sup>3</sup>

d. 382 cm<sup>3</sup>

**Correct Answer:** Option d

2. There are 15 metallic solid identical spherical objects to be loaded on trucks. The diameter of the object is 80 cm. The density of the metal is 7 g/cm<sup>3</sup>. Each truck can carry the maximum load of 6000 kg, what is the greatest number of objects that can be loaded on a truck? (Use  $\pi$  as  $\frac{22}{7}$ )

- a. 3
- b. 4
- c. 5
- d. 6

**Correct Answer:** Option b

**LOB:** Calculate the volume of a given hemisphere in order to infer the quantity of any substance it can hold

1. A hemispherical bowl of diameter 21 cm is completely filled with milk. How many litres of milk, to the nearest tenth, is contained in the bowl? (Use  $\frac{22}{7}$  for  $\pi$ )

- a. 0.2
- b. 1.4
- c. 2.4
- d. 4.9

**Correct Answer:** Option c

2. Fifteen metallic hemispheres of radius 4.5 cm are melted to form 3 identical big hemispheres. If the density of the metal is 8.5 g/cm<sup>3</sup>, what is the mass of a bigger hemisphere? (Use  $\frac{22}{7}$  for  $\pi$ )

- a. 965 g
- b. 8130.63 g
- c. 8138.75 g
- d. 16,261.35 g

**Correct Answer:** Option b

# Suggested Teacher Resources



## Lesson Plan



Objective	Students will be able to find a relation between the side and surface area/volume of a cube.
Prerequisite	Construction of nets of cubes
Materials required	Chart papers, scissors and tapes
Procedure	<ol style="list-style-type: none"> <li>1. Teacher will start the class by dividing the class into 3 groups and giving chart papers, scissors and tapes to all the groups.</li> <li>2. Teacher will first instruct the 1st group to make the net of a cube of side 10cm each.</li> <li>3. Then the teacher will instruct the 2nd group to make the net of a cube having double the side than of the 1st group.</li> <li>4. She will instruct the 3rd group to make the net of a cube having half the side than of the 1st group.</li> <li>5. All the groups will be asked to make the cubes from their nets.</li> <li>6. One representative from each group will present their cube in front of the class.</li> <li>7. Teacher will ask the following questions to the entire class:             <ul style="list-style-type: none"> <li>• Reflect on the change in surface area of the cube when the side of the cube becomes double.</li> <li>• Reflect on the change in surface area of the cube when the side of the cube becomes half.</li> <li>• Reflect on the change in volume of the cube when the side of the cube becomes double.</li> <li>• Reflect on the change in volume of the cube when the side of the cube becomes half.</li> </ul> </li> <li>8. For each of the above pointers, students will be asked to work in their notebooks and come with their answers. They will be asked to generalize their answer for all the values of the sides of the cubes and not be case specific.</li> </ol>
Follow Up Discussion	Teacher can repeat this exercise with other shapes as well- cuboid, cylinder and cone.



## Activity



**Objective:** Students will be able to construct cones with a rectangular piece of paper and analyse its properties.

**Material Required:** Chart papers, tapes and scissors

**Procedure:**

1. Teacher will divide the students in pairs and will give chart paper, scissors and tape to the pairs.
2. Teacher will ask each pair to make 2 identical rectangles of their choice.
3. Teacher will ask the one student to fold the rectangle from its length to make a cone and the other student to fold the rectangle from its breadth to make a cone.
4. Following questions will be asked by the teacher to each pair:
  - What is the difference between the surface areas of both the cones made by the same size of rectangles? Give proper reasoning.

- What is the difference between the volume of both the cones made by the same size of rectangles? Give proper reasoning.
5. Students will be made to work around similar handmade manipulatives of 3d shapes to understand their dimensions and properties.

**Follow Up Discussion:**

Teacher can ask students to make nets of different solids in order to decode the formulas of their surface areas and volumes

# 14. STATISTICS

**QR Code:**



**Learning outcome and Learning Objectives:**

Content area/Concepts	Learning Objectives	Learning Outcome
Frequency Table	Record and label a given data set in order to create a frequency table	Represents given data in different forms like, tabular form (grouped or ungrouped), bar graph, histogram (with equal and varying width and length), and frequency polygon in order to analyse given data
Bar Graph	Identify an appropriate scale and labels in order to represent given data through a bar graph	
	Read a given bar graph in order to infer a variety of information from it	
	Compare the values in order to correlate two data points from the graph	
Histogram	Read the given data in order to create a histogram for continuous and discontinuous data sets	
	Read a given histogram in order to infer a variety of information from it	
Frequency Polygon	Read the given data in order to create a frequency polygon for given data sets	
	Read a given frequency polygon in order to infer a variety of information from it	
Mean, Median and Mode	Differentiate between mean, median and mode with examples in order to understand most effective measure of central tendency in various cases	Identifies daily life situations in order to classify them as situations where mean, median and mode can be used
	Apply appropriate formula in order to calculate the mean and median of even and odd number of data points	
	Recall and use the formula for mean in order find the value of a missing observation	

# Test items



**LOB:** Record and label a given data set in order to create a frequency table

1. Vinod records the ages (in years) of people in a central park as shown below.  
7, 12, 18, 16, 6, 9, 11, 21, 24, 35, 38, 52, 58, 60, 45, 55, 8, 10, 15, 12, 39, 44, 10, 5, 19, 31, 29, 48, 50, 16.  
Which of the frequency tables correctly represents the data with a class size of 8?

a.

Age (in years)	Tally Marks	Number of People
5 - 12		9
13 - 20		6
21 - 28		2
29 - 36		2
37 - 44		4
45 - 52		4
53 - 60		3

b.

Age (in years)	Tally Marks	Number of People
1 - 10		6
11 - 20		9
21 - 30		3
31 - 40		4
41 - 50		5
51 - 60		3

c.

Age (in years)	Tally Marks	Number of People
5 - 12		10
13 - 20		5
21 - 28		2
29 - 36		3
37 - 44		3
45 - 52		4
53 - 60		3

d.

Age (in years)	Tally Marks	Number of People
1 - 10		7
11 - 20		8
21 - 30		3
31 - 40		4
41 - 50		4
51 - 60		4

**Correct Answer:** Option c

2. Riya measured the heights of the plants in her backyard and recorded the data as shown below.  
58.5 cm, 62 cm, 68.1 cm, 52.3 cm, 81.6 cm, 72.5 cm, 69.5 cm, 70.5 cm, 54.1 cm, 55.5 cm, 85.5 cm, 78.3 cm, 82.9 cm, 74.2 cm, 86 cm  
She then incorrectly created a frequency table of the data collected as shown below.

Height of the plant (in cm)	Number of plants
50.5 – 55.5	3
55.5 – 60.5	1
60.5 – 65.5	1
65.5 – 70.5	3
70.5 – 75.5	2
75.5 – 80.5	1
80.5 – 85.5	3
85.5 – 90.5	1

Which of these could be the possible reason of error in Riya's frequency table?

- 70.5 cm and 85.5 cm must be considered in class (70.5 – 75.5) and class (85.5 – 90.5) respectively, and not in class (65.5 – 70.5) and class (80.5 – 85.5).
- 70.5 cm and 85.5 cm must be considered in class (65.5 – 70.5), class (50.5 – 55.5), and class (80.5 – 85.5) respectively, and not in class (70.5 – 75.5) and class (85.5 – 90.5).
- 70.5 cm, 55.5 cm, 85.5 cm must be considered in class (70.5 – 75.5), class (55.5 – 60.5), and class (85.5 – 90.5) respectively, and not in class (65.5 – 70.5), class (50.5 – 55.5), and class (80.5 – 85.5).
- 70.5 cm, 55.5 cm, 85.5 cm must be considered in class (65.5 – 70.5), class (50.5 – 55.5), and class (80.5 – 85.5) respectively, and not in class (70.5 – 75.5), class (55.5 – 60.5), and class (85.5 – 90.5).

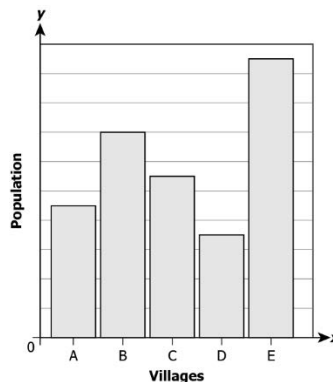
**Correct Answer:** Option c

09M1402: Identify an appropriate scale and labels in order to represent given data through a bar graph

1. A student recorded the population of some villages as shown below.

Village	Population
A	450
B	700
C	550
D	350
E	950

The student then represented the data as shown below.

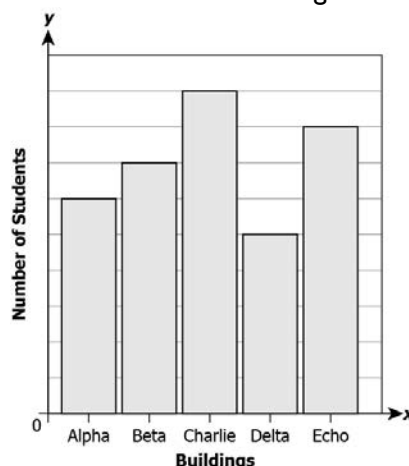


Which of the following would be the scale used on they – axis?

- 1 unit = 10 people
- 1 unit = 50 people
- 1 unit = 100 people
- 1 unit = 500 people

**Correct Answer:** Option c

2. The bar graph below shows the number of students residing at different hostel buildings in a university.



If the total number of students residing in the hostel buildings is 700, how many students reside in Charlie building?

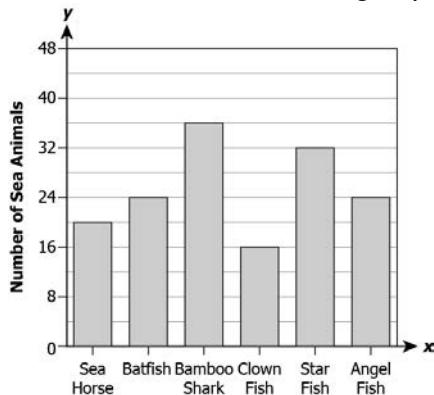
- 90

- b. 135
- c. 180
- d. 225

**Correct Answer:** Option c

**LOB:** Read a given bar graph in order to infer a variety of information from it

1. The bar graph below shows the number of sea animals in a large aquarium.

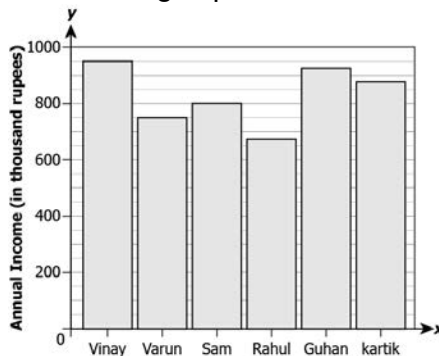


How many more sea horses are there in the aquarium than clown fishes?

- a. 2
- b. 4
- c. 6
- d. 8

**Correct Answer:** Option b

2. The bar graph shows the annual income of a group of friends.



Who earns the most among the group of friends and how much more does he earn than the one who earns the least?

- a. Vinay; Rs 200000
- b. Vinay; Rs 275000
- c. Guhan; Rs 175000
- d. Guhan; Rs 250000

**Correct Answer:** Option b

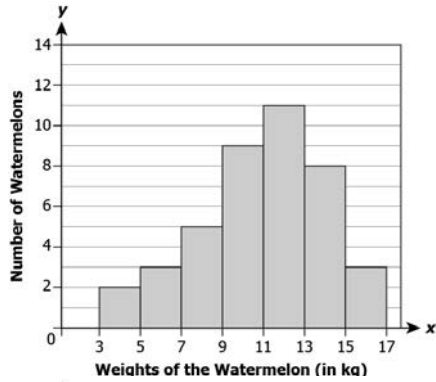
**LOB:** Read the given data in order to create a histogram for continuous and discontinuous data sets

1. The table below show the weights of watermelons at a store.

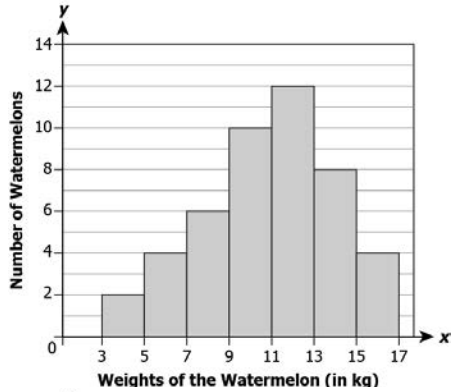
Weights of the watermelon (in kg)	Number of watermelons
3 – 5	2
5 – 7	3
7 – 9	5
9 – 11	9
11 – 13	11
13 – 15	8
15 – 17	3



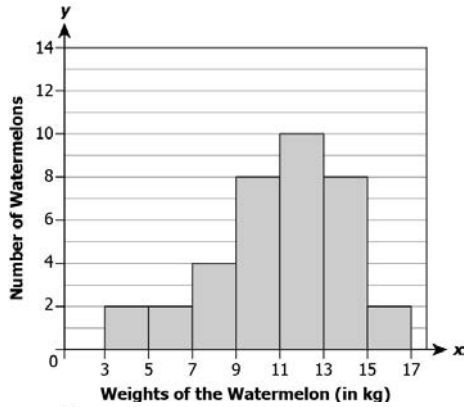
Which of the following histogram represents the given data?



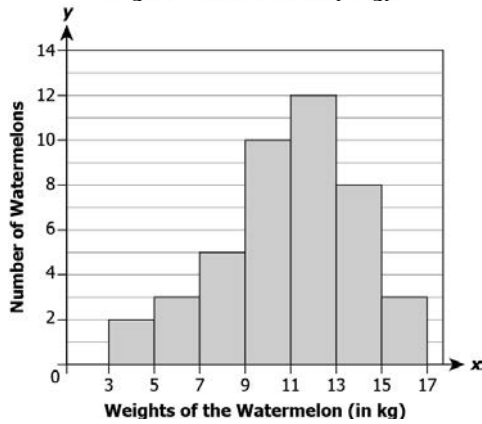
a.



b.



c.



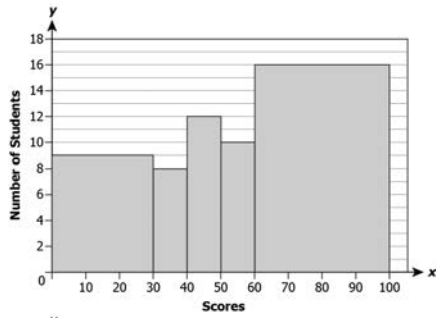
d.

**Correct Answer:** Option a

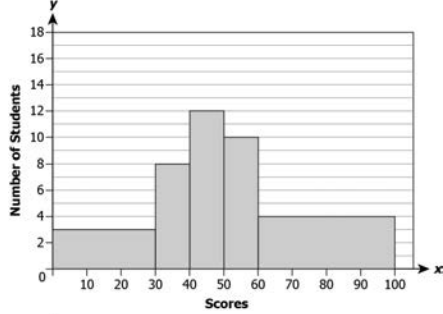
2. The table below shows the quiz scores of a class of students.

Scores	Number of students
Below 30	9
30 – 40	8
40 – 50	12
50 – 60	10
Above 60	16

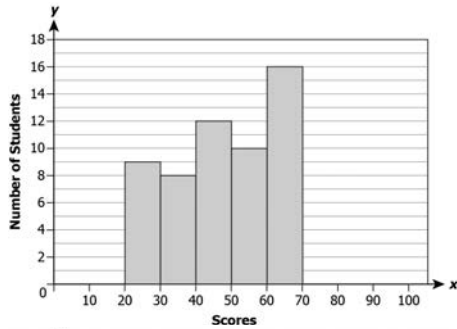
Which of the following histogram represents the data?



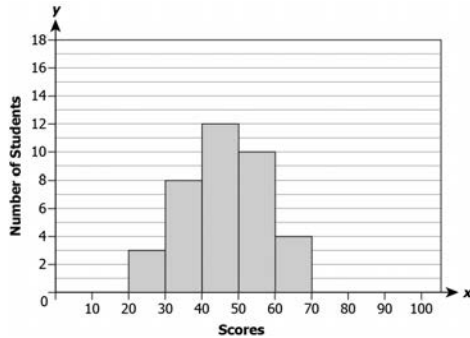
a.



b.



c.

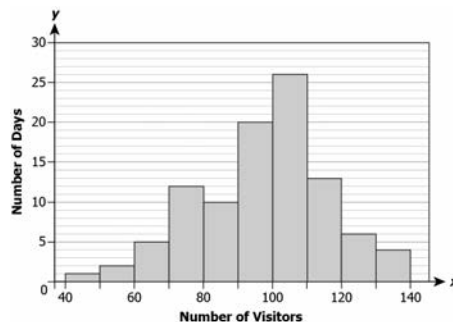


d.

**Correct Answer:** Option b

**LOB:** Read a given histogram in order to infer a variety of information from it

I. The histogram below shows the number of visitors in a museum on different number of days:

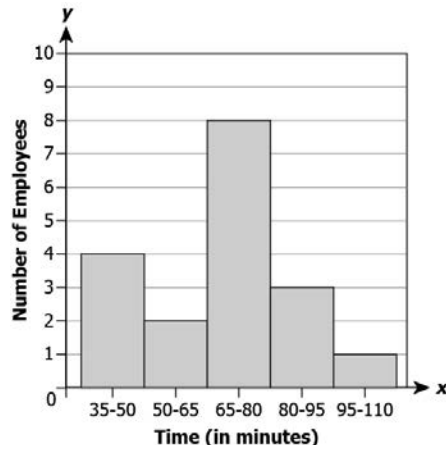


Which of these is correct about the histogram?

- a. There were about 80-90 visitors for 12 days at the museum.
- b. There were about 60-70 visitors for 5 days at the museum.
- c. There were about 120-140 visitors for 6 days at the museum.
- d. There were about 100-120 visitors for 26 days at the museum.

**Correct Answer:** Option b

2. The histogram below shows the daily commute time, in minutes, for 18 employees of an office.



Which of these is NOT correct about the histogram?

- a. 4 employees take 35-50 minutes to commute to office.
- b. 8 employees take 65-95 minutes to commute to office.
- c. 6 employees take less than 65 minutes to commute to office.
- d. 4 employees take more than 80 minutes to commute to office.

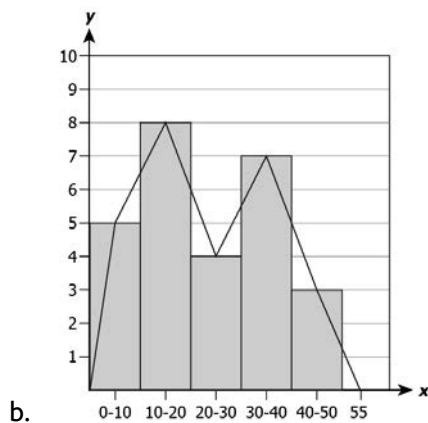
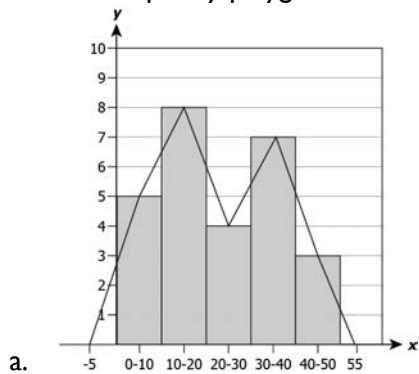
**Correct Answer:** Option b

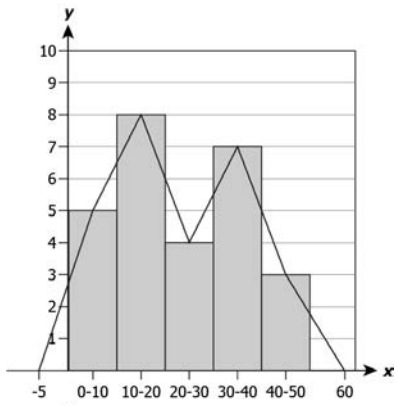
**LOB:** Read the given data in order to create a frequency polygon for given data sets

1. Consider the data set shown below:

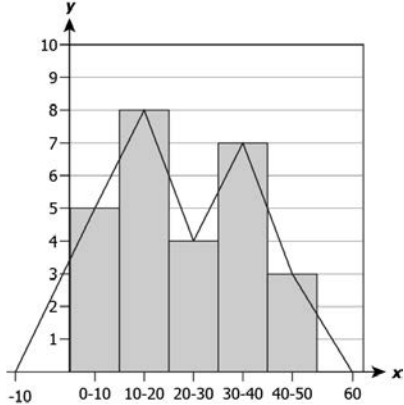
Class	Frequency
0-10	5
10-20	8
20-30	4
30-40	7
40-50	3

Which of these frequency polygons correctly represents the data shown?





c.



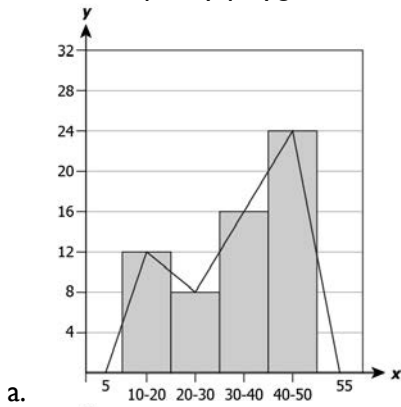
d.

**Correct Answer:** Option a

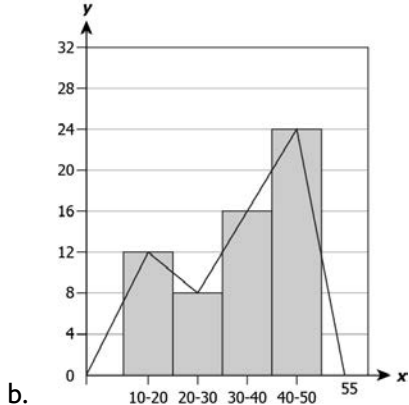
2. The table below shows the points scored by 60 players.

Points	Number of players
10-20	12
20-30	8
30-40	16
40-50	24

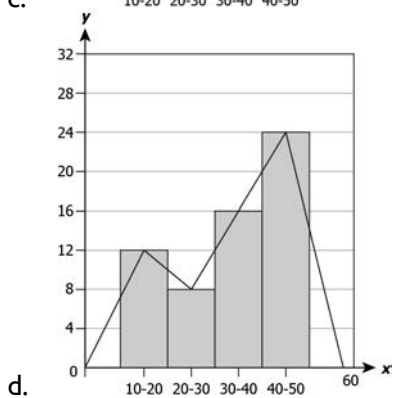
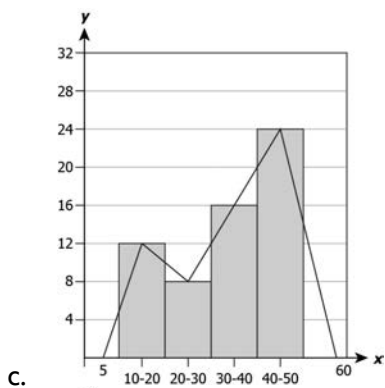
Which of these frequency polygons correctly shows the given data?



a.



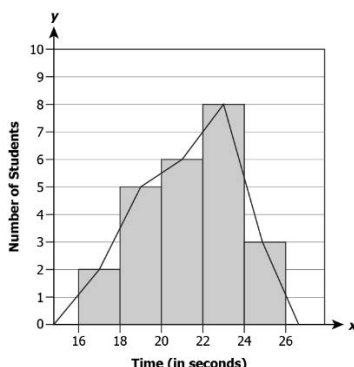
b.



**Correct Answer:** Option a

**LOB:** Read a given frequency polygon in order to infer a variety of information from it

- The frequency polygon below shows the time, in seconds, taken by 24 students of a class to complete a 100-m dash.

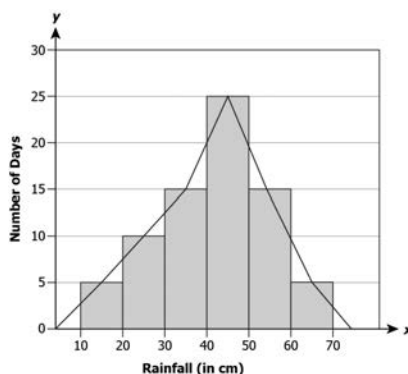


Which of these is NOT true about the data shown in the frequency polygon?

- 11 students took 22 or more minutes to complete the 100 m dash.
- 2 students took 16-18 seconds to complete the 100 m dash.
- 7 students took 16-20 seconds to complete the 100 m dash.
- 8 students took 22-26 seconds to complete the 100 m dash.

**Correct Answer:** Option d

- The frequency polygon below shows the amount, in cm, of rainfall received on different number of days in a city over a year.



Which of these is true about the frequency polygon?

- a. The maximum amount of rainfall received is between 40-50 cm.
- b. The amount of rainfall received was about 10-20 cm for 10 of the days.
- c. The amount of rainfall received was about 50-60 cm for more than 15 of the days.
- d. There were equal number of days when the rainfall was between 10-20 cm and 60-70 cm.

**Correct Answer:** Option d

**LOB:** Differentiate between mean, median and mode with examples in order to understand most effective measure of central tendency in various cases

1. The amount of snowfall received over two weeks in a city is listed below.  
2 cm, 3 cm, 6 cm, 2 cm, 4 cm, 3 cm, 12 cm, 5 cm, 1 cm, 3 cm, 4 cm, 3 cm, 2 cm, 6 cm

Which of the following is the most effective measure of central tendency?

- a. Mean because the data has extreme data points.
- b. Median because the data has extreme data points.
- c. Mean because the data has no extreme data points.
- d. Median because the data has no extreme data points.

**Correct Answer:** Option b

2. There are 16 students in a dance class. The age of the students (in years) are shown below.  
12, 14, 18, 20, 11, 13, 22, 24, 14, 19, 20, 16, 12, 14, 20, 15.

If two more students of age 10 and 14 years join the class, which statement is true about the central tendency of the data?

- a. The central tendency of the data decreases by 1 as the mean decreases by 1.
- b. The central tendency of the data decreases by 1 as the median decreases by 1.
- c. The central tendency of the data decreases by 0.5 as the mean decreases by 0.5.
- d. The central tendency of the data decreases by 0.5 as the median decreases by 0.5.

**Correct Answer:** Option c

**LOB:** Apply appropriate formula in order to calculate the mean and median of even and odd number of data points

1. What is the mean and median of the data set shown below?

10, 5, 6, 2, 11, 13, 5, 8, 3

- a. Mean: 7; Median: 5
- b. Mean: 7; Median: 6
- c. Mean: 6; Median: 7
- d. Mean: 6; Median: 6

**Correct Answer:** Option b

2. The data set below shows the time, in minutes, taken by 10 students to solve a mathematics problem.  
2, 5, 8, 4, 4, 3, 6, 2, 11, 5

What is the mean and median of the time taken?

- a. The mean time is 5 minutes and the median time is 4.5 minutes.
- b. The mean time is 4.5 minutes and the median time is 5 minutes.
- c. The mean time is 5 minutes and the median time is 4 minutes.
- d. The mean time is 4 minutes and the median time is 5 minutes.

**Correct Answer:** Option a

**LOB:** Recall and use the formula for mean in order find the value of a missing observation

1. Consider the data set shown below.

12,  $x$ , 10, 4, 7, 8, 7, 5

If the mean of the given data set is 7, what is the value of  $x$ ?

- a. 3
- b. 4
- c. 7
- d. 10

**Correct Answer:** Option a

2. In the table below, the grocery expenditure of 2 months of a family is missing.

Month	March	April	May	June	July	August
Expenditure (in ₹)	2500	3400	?	?	5100	2800

The average grocery expenditure of the given 6 months is ₹3250. The expenditure in June is half of the expenditure in May. Which of these shows the grocery expenditure of the family for May and June?

- a. The family spend ₹3800 in May and ₹1900 in June.
- b. The family spend ₹1900 in May and ₹3800 in June.
- c. The family spend ₹3800 in May and ₹7600 in June.
- d. The family spend ₹1900 in May and ₹950 in June.

**Correct Answer:** Option a

# Suggested Teacher Resources



## Lesson Plan



<b>Objectives</b>	<ul style="list-style-type: none"> <li>• Collect data for an activity given by the teacher and compare with peers in order to infer that the way data is collected, organized and displayed influences interpretation.</li> <li>• Record and label a given data set in order to create a frequency table</li> </ul>
<b>Prerequisite Knowledge</b>	Frequency, Tally Marks
<b>Material Required</b>	Pen and Paper
<b>Procedure</b>	<p>Divide the class into three groups and ask the students of each group to collect data for an activity given by the teacher –</p> <p>The data collection instructions for the groups are –</p> <ol style="list-style-type: none"> <li>1. Group 1 will collect the data of the Month of Birth (Jan, Feb, March.... December) of each of the group's member.</li> <li>2. Group 2 will collect the data of the Marks obtained in the last test out of 10 (e.g. 1,2,3,4....10) of each of the group's member.</li> <li>3. Group 3 will collect the data of the Favorite Sport (Football, Hockey, Badminton etc.) for each of the group's member.</li> </ol> <p>Now ask the students to compare with peers in order to infer that the way data is collected, organized and displayed.</p> <p>Now put the following questions in front of the whole class and take large group responses and write them on board!</p> <ul style="list-style-type: none"> <li>• What are the trends that are visible in these data?</li> <li>• What are some similarities in the form of collection of data?</li> <li>• What are some dis-similarities in collection of data?</li> </ul> <p>Now give the student a task to organize the data in a table-</p> <p>Sample responses for the three groups will be –</p> <p><b>Group-I</b></p>



Month	Students Born
Jan	3
Feb	4
March	5
April	7
May	2
June	1
July	3
August	1
Sept	2
Oct	2
Nov	3
Dec	5

Data



**Group -2**

Marks	Number of Students = Frequency	Tally Marks
1	1	
2	4	
3	2	
4	5	
5	7	
6	4	
7	4	
8	6	
9	1	
10	1	
Total	35	

**Group – 3**

Sports	Number of Students = frequency	Tally Marks
Badminton	5	
Cricket	10	
Football	4	
Hockey	1	
Tennis	3	
Volleyball	7	

4. Teacher will then explain the meaning of a class, frequency, frequency distribution to the students.

5. Teacher will conclude the class by asking the students various situations where data collection and representation is necessary.

Reflection Questions

- Why is data collection necessary in real life?
- What are the different ways of organizing data?

Expected Outcome

Student will learn about-

- Different ways of data collection.
- Importance of Data collection.
- Frequency Distribution Table.

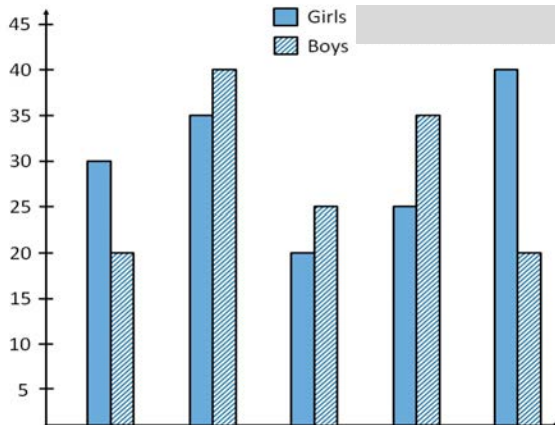
2

**Activity**

Objective: Students would be able to represent data in a double bar graph

Procedure:

1. Teacher will divide the students in groups of 5.
2. She will ask each group to go to different sections of class 9 and ask the teacher the number of girls and boys present in the class.
3. Once the students return back to their class, representative of each group will come to the blackboard and draw double bar graph representing number of girls and boys in each section of the 9th class.



4. Follow up questions:
  - Which section of class 9 has maximum boys?
  - Which section of class 9 has minimum girls?
  - What is the difference between the maximum student strength and minimum student strength?

# 15. PROBABILITY

**QR Code:**



**Learning outcome and Learning Objectives:**

Content area/Concepts	Learning Objectives	Learning Outcome
Calculation of Empirical Probability in various experiments	Recall the formula for Empirical probability to calculate the probability for a simple event	Conducts experiments and analyses data in order to calculate empirical probability
	Create a flow chart of all the terms related to random experiments (coins, dice, cards) in order to calculate the total number of trials of a given experiment and calculate the Empirical Probability	
	Compute the total number of trials and trials for a given event E represent in various forms (table, histogram, pie-charts, etc) to solve for the value of Empirical Probability P(E)	
	Calculate empirical probability of a situation in order to predict the likelihood of an event	
	Arrange events from least likely to most likely in order to predict outcomes in a given experiment	
	Calculate the sum of probabilities of all events in order to Prove that the sum of the probability of all events in a single experiment is 1	

# Test items



**LOB:** Recall the formula for Empirical probability to calculate the probability for a simple event

1. A bottle has some red, blue, yellow, and green marbles. Anjali picks each marble 100 times and records her observation as shown.

Red marble	24
Blue marble	39
Yellow marble	22
Green marble	15

What is the probability of picking a red marble from the bottle the next time?

- a.  $\frac{\text{Number of times red marble is picked}}{\text{Total number of times Anjali picks the marble}}$
- b.  $\frac{\text{Total number of times Anjali picks the marble}}{\text{Number of times red marble is picked}}$
- c.  $1 - \frac{\text{Number of times red marble is picked}}{\text{Total number of times Anjali picks the marble}}$
- d.  $1 - \frac{\text{Total number of times Anjali picks the marble}}{\text{Number of times red marble is picked}}$

**Correct Answer:** Option a

2. A die is rolled 150 times and each outcome is recorded as shown.

<b>Outcome</b>	1	2	3	4	5	6
<b>Frequency</b>	30	35	28	15	22	20

What is the probability of the die landing on an even number that is not a multiple of 4 next time?

- a.  $\frac{\text{Frequency of 2}}{\text{Total number of times the die is rolled}}$
- b.  $\frac{\text{Frequency of 2} + \text{Frequency of 6}}{\text{Total number of times the die is rolled}}$
- c.  $1 - \frac{\text{Frequency of 2} + \text{Frequency of 6}}{\text{Total number of times the die is rolled}}$
- d.  $\frac{\text{Frequency of 2} + \text{Frequency of 4} + \text{Frequency of 6}}{\text{Total number of times the die is rolled}}$

**Correct Answer:** Option b

**LOB:** Create a flow chart of all the terms related to random experiments (coins, dice, cards) in order to calculate the total number of trials of a given experiment and calculate the Empirical Probability

1. Which option shows all possible outcomes when a die and a coin are tossed simultaneously?
- a.  $\{(1, T), (2, T), (3, T), (4, T), (5, T), (6, T)\}$
- b.  $\{(1, H), (2, H), (3, H), (4, H), (5, H), (6, H)\}$
- c.  $\{(1, H), (1, T), (2, H), (2, T), (3, H), (3, T), (4, H), (4, T)\}$
- d.  $\{(1, H), (1, T), (2, H), (2, T), (3, H), (3, T), (4, H), (4, T), (5, H), (5, T), (6, H), (6, T)\}$

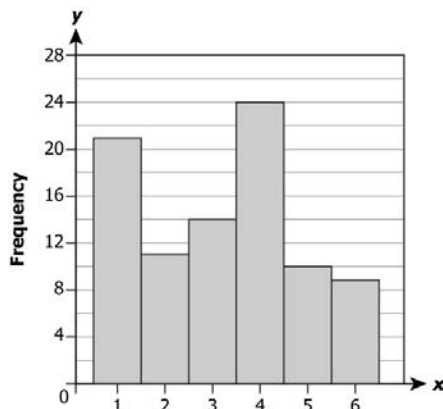
**Correct Answer:** Option d

2. A teacher was asked to select four students for an inter-school quiz competition. Based on the gender, (boy or girl), the teacher lists all the possibilities of selecting the students as L1. She then selects one boy as the first candidate and lists the possibilities for selecting other three students as L2. Which of the following is true?
- a. Number of possibilities in L1 is twice the number of possibilities in L2.
- b. Number of possibilities in L1 is thrice the number of possibilities in L2.
- c. Number of possibilities in L2 is twice the number of possibilities in L1.
- d. Number of possibilities in L2 is thrice the number of possibilities in L1.

**Correct Answer:** Option a

**LOB:** Compute the total number of trials and trials for a given event E represent in various forms (table, histogram, pie-charts, etc.) to solve for the value of Empirical Probability P(E)

1. Anita rolled a dice  $n$  number of times and records her observation as shown.

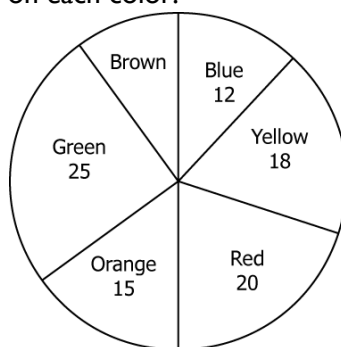


Which of the following gives the number of times Anita rolls the dice?

- a.  $(21 + 12 + 14 + 24 + 10 + 9)$
- b.  $(20 + 10 + 14 + 24 + 10 + 8)$
- c.  $(21 + 11 + 14 + 24 + 10 + 9)$
- d.  $(22 + 12 + 14 + 24 + 10 + 10)$

**Correct Answer:** Option c

2. Karan has a spinner with 6 colors. He spins the spinner  $n$  number of times. The pie chart below shows the number of times the spinner landed on each color.



If the probability of the spinner landing on blue colour is the same as the probability of the spinner landing on brown, then how many times did Karan spin the spinner?

- a. 90
- b. 96
- c. 100
- d. 102

**Correct Answer:** Option d

**LOB:** Calculate empirical probability of a situation in order to predict the likelihood of an event

1. A survey was conducted on 140 people at random who visited a mall about the number of pets they own. The results of the survey are recorded as shown.

Number of pets	0	1	2	3
Number of people	64	38	26	12

What is the probability of selecting a person in the mall having more than 2 pets?

- a.  $\frac{12}{140}$
- b.  $\frac{26}{140}$
- c.  $\frac{140}{2+3}$
- d.  $\frac{140}{26+12}$

**Correct Answer:** Option d

2. There are tickets numbered 1 through 8. A ticket is randomly drawn and the number on it is recorded. This experiment is repeated 160 times. The table shows the result of the experiment.

Number on Ticket	1	2	3	4	5	6	7	8
Frequency	6	18	32	35	15	14	12	28

If the experiment is repeated 200 times, how many times is the ticket numbered 7 expected to be drawn?

- a. 12
- b. 15
- c. 17
- d. 20

**Correct Answer:** Option b

**LOB:** Arrange events from least likely to most likely in order to predict outcomes in a given experiment

1. The birth month of 50 students of a class are recorded as shown.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Number of Students	5	2	4	5	4	6	4	8	3	1	5	3

Which of the following is true when a student is randomly selected from the class?

- a. A student who is born in August is more likely to be selected than a student who is born in March, as  $0.16 > 0.08$ .
- b. A student who is born in March is more likely to be selected than a student who is born in August, as  $0.16 > 0.08$ .
- c. A student who is born in August is more likely to be selected than a student who is born in March, as  $0.8 > 0.4$ .
- d. A student who is born in March is more likely to be selected than a student who is born in August, as  $0.8 > 0.4$ .

**Correct Answer:** Option a

2. Alex randomly asked 150 students at a university about the number of vehicles they have at their residence. He noted the results as shown.

Number of vehicles	0	1	2	3
Number of students	15	42	54	39

If  $P(A)$ ,  $P(B)$ ,  $P(C)$ ,  $P(D)$  represent the probability of randomly selecting a student who has 0, 1, 2, 3 vehicles respectively, which of the following shows the increasing order of the probabilities?

- a.  $P(A)$ ,  $P(D)$ ,  $P(B)$ ,  $P(C)$ , as  $P(A) = 0.1$ ,  $P(B) = 0.28$ ,  $P(C) = 0.36$ , and  $P(D) = 0.26$ .
- b.  $P(C)$ ,  $P(B)$ ,  $P(D)$ ,  $P(A)$ , as  $P(A) = 0.1$ ,  $P(B) = 0.28$ ,  $P(C) = 0.36$ , and  $P(D) = 0.26$ .
- c.  $P(A)$ ,  $P(D)$ ,  $P(B)$ ,  $P(C)$ , as  $P(A) = 0.15$ ,  $P(B) = 0.42$ ,  $P(C) = 0.54$ , and  $P(D) = 0.39$ .
- d.  $P(C)$ ,  $P(B)$ ,  $P(D)$ ,  $P(A)$ , as  $P(A) = 0.15$ ,  $P(B) = 0.42$ ,  $P(C) = 0.54$ , and  $P(D) = 0.39$ .

**Correct Answer:** Option a

**LOB:** Calculate the sum of probabilities of all events in order to Prove that the sum of the probability of all events in a single experiment is 1

1. Santosh picked a card from a deck of 52 cards and noted the symbol on the card. He repeated this experiment 120 times and records his observation in the table shown.

Card	Heart	Spade	Diamond	Club
Frequency	36	21	33	30

Which of the following shows that the sum of probabilities of all events (getting a heart, getting a spade, getting a diamond and getting a club) is equal to 1?

- a.  $0.36 + 0.21 + 0.33 + 0.30 = 1$
- b.  $0.3 + 0.175 + 0.275 + 0.25 = 1$
- c.  $0.3 + 0.075 + 0.375 + 0.25 = 1$
- d.  $0.2 + 0.175 + 0.275 + 0.35 = 1$

**Correct Answer:** Option b

2. Virat asks 80 students at a sports club about their favorite sport. He notes his reading as shown.

Favorite Sport	Cricket	Football	Basketball	Tennis
Number of Students	24	18	12	26

He finds the probability of randomly selecting a student who likes cricket, football, basketball, tennis individually and claims that their sum is less than 1. Is he correct?

- a. No, since  $0.48 + 0.36 + 0.24 + 0.52 > 1$ .
- b. Yes, since  $0.24 + 0.18 + 0.12 + 0.26 < 1$ .
- c. No, since  $0.3 + 0.225 + 0.15 + 0.325 = 1$ .
- d. Yes, since  $0.15 + 0.1125 + 0.075 + 0.1625 < 1$

**Correct Answer:** Option c

# Suggested Teacher Resources



## Lesson Plan



Objective	Students will be able to recall the formula for Empirical probability to calculate the probability for a simple event												
Prerequisite	Fraction division and multiplication												
Vocabulary words	Probability, empirical probability, trails, event												
Procedure	<p>Teacher will start the class by asking if students have heard the following sentences:</p> <ul style="list-style-type: none"> <li>● It will probably rain today.</li> <li>● I doubt that he will pass the test.</li> <li>● Most probably, Kavita will stand first in the annual examination.</li> <li>● Chances are high that the prices of diesel will go up.</li> <li>● There is a 50-50 chance of India winning a toss in today's match.</li> </ul> <ol style="list-style-type: none"> <li>1. She will initiate the discussion: What does words 'probably', 'doubt', 'probably', 'chances' point towards? Ans - uncertainty or surety about any event</li> <li>2. Teacher will then introduce the definition of Probability and explain its importance in our life.</li> <li>3. Now, student will divide the class in 4 groups. One group will get a coin. Second group will get 2 coins. Third group will get a dice and the fourth group will get 2 dice.</li> <li>4. Teacher will instruct the 1st and 2nd groups to toss the coin/coins 10 times and write the number of tails and heads after each toss.</li> </ol> <table border="1" style="margin: 10px auto;"> <thead> <tr> <th>Number of times the coin is tossed</th> <th>Number of times tails come up</th> <th>Number of times heads come up</th> </tr> </thead> <tbody> <tr> <td>10</td> <td></td> <td></td> </tr> </tbody> </table> <table border="1" style="margin: 10px auto;"> <thead> <tr> <th>Number of times 2 coins are tossed</th> <th>Number of times tails come up</th> <th>Number of times heads come up</th> </tr> </thead> <tbody> <tr> <td>10</td> <td></td> <td></td> </tr> </tbody> </table> <p>Now students will write the probability of tails coming up and heads coming up. Toss the coin twenty times and in the same way record your observations as above. Again, find the probability for this collection of observations. Repeat the same experiment by increasing the number of tosses and record the number of heads and tails. Then find the probability of the same.</p> <ol style="list-style-type: none"> <li>5. Teacher will conclude that as the number of tosses gets larger, the values of the probability of head/tail come closer to <math>1/2</math>.</li> <li>6. Similarly, the teacher will instruct the 3rd and 4th groups to roll the dice/2 dice 10 times and write the number of times digit 1,2,3,4,5,6 comes at the top.</li> </ol>	Number of times the coin is tossed	Number of times tails come up	Number of times heads come up	10			Number of times 2 coins are tossed	Number of times tails come up	Number of times heads come up	10		
Number of times the coin is tossed	Number of times tails come up	Number of times heads come up											
10													
Number of times 2 coins are tossed	Number of times tails come up	Number of times heads come up											
10													

Number of times the dice is rolled	Number of times digit 1 comes up	Number of times digit 2 comes up	Number of times digit 3 comes up	Number of times digit 4 comes up	Number of times digit 5 comes up	Number of times digit 6 comes up
10						

Number of times 2 dice are rolled	Number of times digit 1 comes up	Number of times digit 2 comes up	Number of times digit 3 comes up	Number of times digit 4 comes up	Number of times digit 5 comes up	Number of times digit 6 comes up
10						

Now students will write the probability of digits 1,2,3,4,5,6 coming up. Roll the dice twenty times and in the same way record your observations as above. Again, find the probability for this collection of observations.

Repeat the same experiment by increasing the number of rolls and record the number of 1,2,3,4,5,6 comes up. Then find the probability of the same.

- Teacher will conclude that as the number of rolls gets larger, the values of the probability of 1/2/3/4/5/6 come closer to 1/6.
- At the end of the lesson, teacher will ask students to repeat such experiments with more than 2 coins/dice.

2

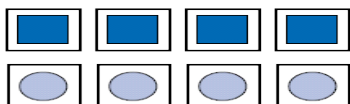
## Activity



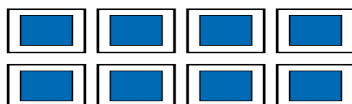
**Objective:** Students will be able to define most likely, equally likely events.

**Material Required:** Square cards and circle cards.

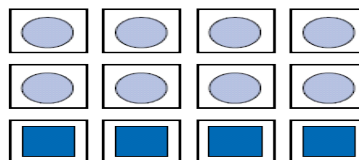
Cards for game 1



Cards for game 2



Cards for game 3



**Procedure:**

- The class is going to play three games.
- In each game some cards are put into a bag.
- Each card has a square or a circle on it.
- One card will be taken out, then put back. If it is a circle, the girls score a point. If it is a square, the boys score a point.
- After playing all the games, Teacher will discuss following questions with the students:
  - Which game are the girls most likely to win? Why?
  - Which game is it equally likely that the boys or girls win? Why?
  - Are any of the games unfair? Why?

**Follow Up Questions:**

- Teacher will ask the students to give examples of events which are equally likely to happen.
- Teacher will ask the students to identify games from their surroundings which have fair chances of winning for each player.



